# Chapter 4

# Inequalities

# Section 4.1
# Inequalities

Unlike equations, inequalities do not provide an exact answer to a problem. Inequalities instead tell us that the solution could be one number (any number) within a set of numbers.

For example, the equation

$$Profit = Price - Cost$$

helps us find the exact profit if the price and cost of the item is given. In plain English:

*If an item costs $80, and is sold for $90, the profit is $10.*

However, if we want to say “I want to make some money”, then the inequality

$$Sales > Costs$$

represents a better mathematical relationship: *My costs are* $250. *Sales* > 250

Four symbols are used to show inequalities:

- $>$ to show an amount “greater than”
- $<$ to show an amount “less than”
- $\geq$ to show an amount “greater than or equal to”
- $\leq$ to show an amount “less than or equal to”

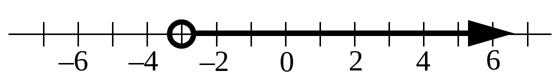
**Examples**:

1. $x > -3$ reads “$x$ is greater than –3” (Answer: –2, –1, 0, 1, 2...)
2. $5 > y$ reads “$y$ is less than 5” (Answer: 4, 3, 2...)
3. $a \geq 0$ reads “$a$ is greater than or equal to 0” (Answer: 0, 1, 2, 3...)
4. $-2 \geq b$ reads “$b$ is less than or equal to –2” (Answer: –2, –3, –4...)

**GRAPHING INEQUALITIES**

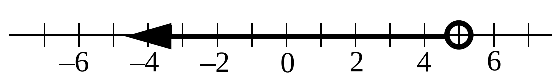
Inequalities are graphed as arrows that show the direction of the solution.

The graph of example 1 above is:

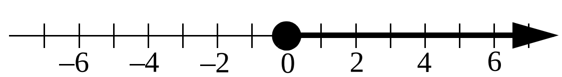



The circle around –3 means the solution does NOT reach –3.

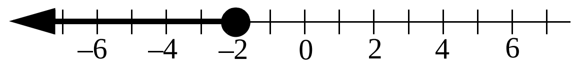
The graph of example 2 above is:




The graph of example 3 above is:




Unlike the empty circle above, a filled circle means “0” is a solution.

The graph of example 4 above is: 

Notice the difference in plotting “greater than” and “greater than or equal.” When *greater than* or *less than* are plotted, the circle marking the limit of the answer is open; when “equal” is added, the circle is filled.

**Practice**:

CONTENTS OF THIS PAGE HAVE BEEN REMOVED TO PROTECT COPYRIGHT

## ADDING AND SUBTRACTING INEQUALITIES

Inequalities are solved using the same inverse rules we use in solving equations; however, **inequalities change direction when we multiply or divide both sides of the inequality by a negative number**.

**Example**:

$$x + 5 > 7 \quad \text{subtract 5 from both sides}$$
$$x + 5 - 5 > 7 - 5$$
$$x > 2$$

**Example**:

$$3x + 8 < 4x - 7 \quad \text{subtract 8 from both sides}$$
$$3x + 8 - 8 < 4x - 7 - 8$$
$$3x < 4x - 15 \quad \text{subtract 4x from both sides}$$
$$3x - 4x < 4x - 4x - 15$$
$$-x < -15 \quad \text{divide by } -1 \text{ to make x positive}$$
$$\frac{-x}{-1} < \frac{-15}{-1}$$
$$x > 15$$

direction of inequality changed when *x* turned positive

## MULTIPLYING AND DIVIDING INEQUALITIES

**Example**:

$$3(x - 2) - 5x < 24 \quad \text{distribute parenthesis}$$
$$3x - 6 - 5x < 24 \quad \text{combine like terms}$$
$$-2x < 24 + 6$$
$$-2x < 30 \quad \text{divide by } -2$$
$$\frac{-2x}{-2} < \frac{30}{-2}$$
$$x > -15 \quad \text{change direction}$$

**Example**:

$$\frac{4x}{5} \geq \frac{8}{9}$$
$$\frac{5(4x)}{5} \geq \frac{8(5)}{9}$$

multiply both sides by 5 to remove 5 from left side

$$4x \geq \frac{40}{9}$$
$$\frac{4x}{4} \geq \frac{40}{9(4)}$$

divide both sides by 4 to remove 4 from left side

$x \geq \frac{40}{36}$ reduce fraction to $x \geq \frac{10}{9}$

**Example**: A student's average for 9 tests is 84 points. What is the lowest score he can achieve on a tenth test to raise his average above 85 points? (Average > 85)

Make L the lowest score he could get. If the total number of points for 9 tests is

$$9 \times 84 = 756$$

Then the total number of points for 10 tests is 756 + L and the equation

$$\frac{756 + L}{10} > 85 \quad \text{defines the new average.}$$

Solving for L

$$756 + L > 850$$
$$L > 850 - 756$$
$$L > 94$$

The lowest score is 95. Any score greater than 94 will raise the average above 85 points.

**Practice**:

CONTENTS OF THIS PAGE HAVE BEEN REMOVED TO PROTECT COPYRIGHT

CONTENTS OF THIS PAGE HAVE BEEN REMOVED TO PROTECT COPYRIGHT

# Section 4.2
# Absolute Values

A value becomes absolute if we decide to disregard the opposite (negative) nature of numbers. In practice all absolute values become positive.
Absolute values are always shown surrounded by two vertical bars.

**Examples**:
1. $|9| = 9$
2. $|-9| = 9$
3. $|-14 + 6| = |-8| = 8$
4. $|-7 + 23| = |16| = 16$
5. $-18 + |-5 + 2| = -18 + |-3| = -18 + 3 = -15$

NOTE: (In 5 above, because –18 is not absolute—no bars—the answer is negative. Only –3 turns positive).

**EQUATIONS WITH ABSOLUTE VALUES**

Because the answer to an absolute expression is never negative, we then must realize that each unknown absolute value has the possibility of coming from either a positive or a negative value, producing then two solutions.

**Example**: In $|x| = 8$ the value for $x$ could be 8 or –8

One possible answer is positive and one is negative.

**Example**:

$$|y| + 9 = 14$$
$$|y| = 14 - 9$$
$$|y| = 5$$
$$y = 5 \quad y = -5$$

**Example**: $|5x - 7| = 8$

| positive outcome | | negative outcome |
|---|---|---|
| $|5x - 7| = 8$ | | $|5x - 7| = -8$ |
| $5x - 7 = 8$ | | $5x - 7 = -8$ |
| $5x = 8 + 7$ | | $5x = -8 + 7$ |
| $5x = 15$ | | $5x = -1$ |
| $x = 3$ | or | $x = \frac{-1}{5}$ |

**ABSOLUTE EQUATIONS EQUAL TO NEGATIVE OUTCOMES CANNOT BE SOLVED.**

**Examples**:

1. $|5x - 7| = -8$ (No solution. Absolute value cannot be negative).
2. $|5x - 7| + 10 = 8$ (Right side becomes $-10 + 8 = -2$. No solution).

**Practice**:

CONTENTS OF THIS PAGE HAVE BEEN
REMOVED TO PROTECT COPYRIGHT

**INEQUALITIES WITH ABSOLUTE VALUES**

Inequalities give a "range" of values. When inequalities include absolute expressions,

a **conjunction** ($|x| < 5$) or a **disjunction** ($|x| > 4$) is formed**.**

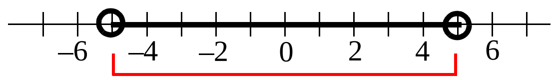
In a **conjunction** (sometimes defined as “in between sets”), two values emerge. They become external limits (for example, a fixed set of integers) beyond which answers will not be found. In the conjunction

$$|x| < 5$$

the answer yields two boundaries $x < 5$ and $x > -5$ (less than “+”, greater than “–”)

or $-5 < x < 5$

graphing it



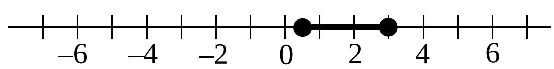

**Example**: $|4x - 7| \le 5$

| positive possibility | negative possibility |
|---|---|
| $4x - 7 \le 5$ | $4x - 7 \ge -5$ |
| $4x \le 7 + 5$ | $4x \ge 7 - 5$ |
| $4x \le 12$ | $4x \ge 2$ |
| $x \le \frac{12}{4}$ | $x \ge \frac{2}{4}$ |
| $x \le 3$ | $x \ge \frac{1}{2}$ |

in between set

*or* $\frac{1}{2} \le x \le 3$

graphing it




In a **disjunction** (sometimes defined as “beyond sets”), the values are limitless in two different directions, one positive, one negative. For example, in the disjunction

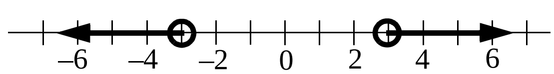
$$|x| > 3$$

the answer sets are in two opposite directions

$x > 3$ *and* $x < -3$ (greater than “+”, less than “–”)

$x < -3$ *or* $x > 3$

graphing it




**Example**: $|2x + 5| \ge 6$

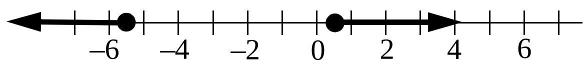

| positive possibility | negative possibility |
|---|---|
| $2x + 5 \ge 6$ | $2x + 5 \le -6$ |

$$2x \geq 6 - 5 \qquad\qquad 2x \leq -6 - 5$$
$$2x \geq 1 \qquad\qquad 2x \leq -11$$
$$x \geq \frac{1}{2} \qquad\qquad x \leq -\frac{11}{2}$$
$$x \geq 0.5 \qquad\qquad x \leq -5.5$$

$$x \leq -5.5 \quad \textit{or} \quad x \geq 0.5$$

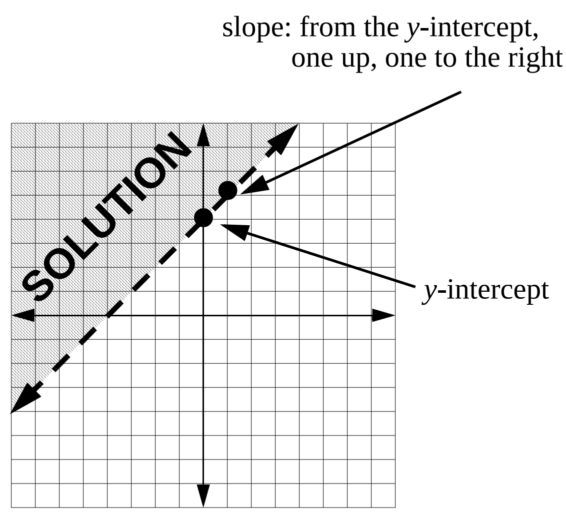
graphing it

**Practice**:



# Section 4.3
# Graphing Inequalities with Two Variables

Because an inequality does not represent *one* exact answer ($a > -3$), but a definite *set* of many answers, when we try to plot an inequality with two variables, the solution is a region containing many points.

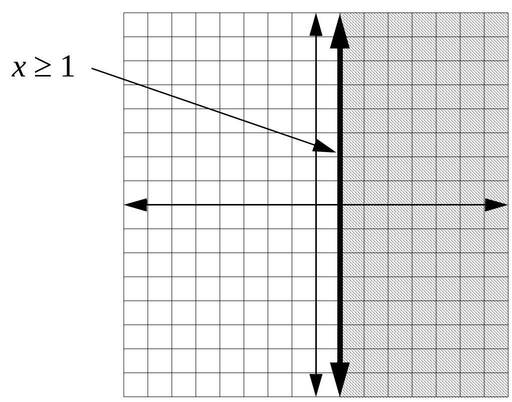



**Example**: Graph the inequality $y > x + 4$

First we plot the boundary line by recognizing that the slope of the line is 1 (coefficient of *x*) and the *y*-intercept is 4 (See graph to the right).

Now we set the region using only the direction given by *y*. According to the example, ($y >$) *y* is "greater", thus the solution area is above the line.

**Example**: Graph the inequality $x \geq 1$

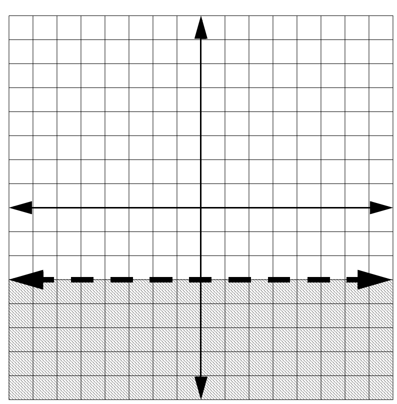



Graphed in two dimensions, an inequality with only one variable is either vertical (*x* only) or horizontal (*y* only). In this case it is vertical passing through $x \geq 1$.

Notice that in the two examples shown above, the line for "greater than" (>) is graphed *dashed* and the line for "greater than or equal to" (≥) is graphed *solid*. Solid indicates that points ON the line are solutions.

**Example**: Graph the inequality $y < -3$

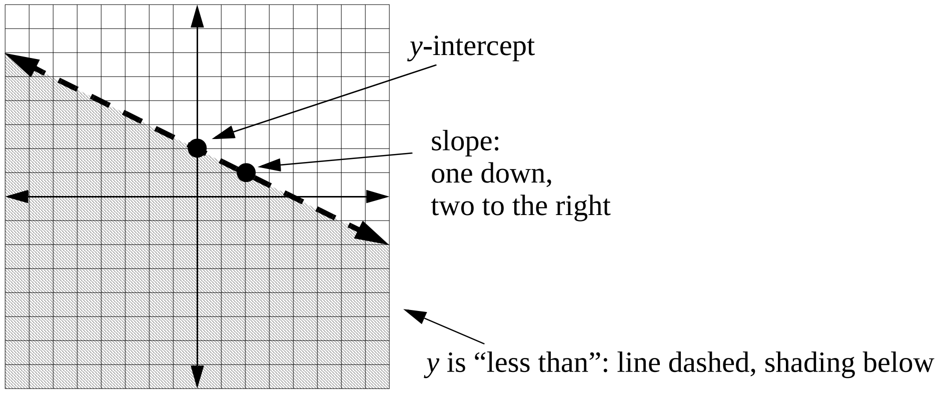
Graphed in two dimensions, an inequality with only one variable is either vertical (*x* only) or horizontal (*y* only). In this case it is horizontal passing through $y < -3$.



Because the *y* in the inequality indicates "less than", the solution area is found below the *dashed* (<) line.

**Example**: Graph the inequality $y < -\frac{1}{2}x + 2$

Because the inequality is already in *y*-intercept form, we can read the *y*-intercept as 2 and the slope as $-\frac{1}{2}$.




**Example**: Graph the inequality $3x + 2y \leq 6$

Because the inequality is not in *y*-intercept form, we first turn it into *y*-intercept by solving for *y*.

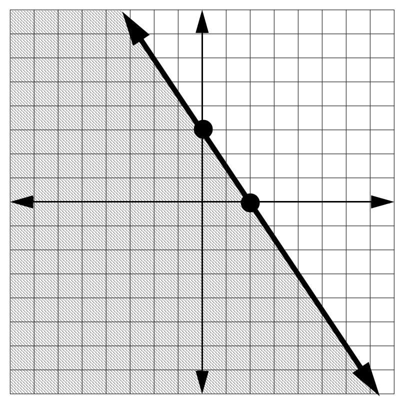
$$3x - 3x + 2y \leq -3x + 6 \quad \text{subtract } 3x$$

$$2y \leq -3x + 6 \quad \text{divide by 2}$$

$$\frac{2y}{2} \leq -\frac{3}{2}x + \frac{6}{2}$$

$$y \leq -\frac{3}{2}x + 3$$

The *y*-intercept is 3 and the slope is $\frac{-3}{2}$.



Because the inequality is $y \leq$ ("less than or equal to"), the area is under the line and the line is solid.

**Practice**:

CONTENTS OF THIS PAGE HAVE BEEN REMOVED TO PROTECT COPYRIGHT

CONTENTS OF THIS PAGE HAVE BEEN REMOVED TO PROTECT COPYRIGHT