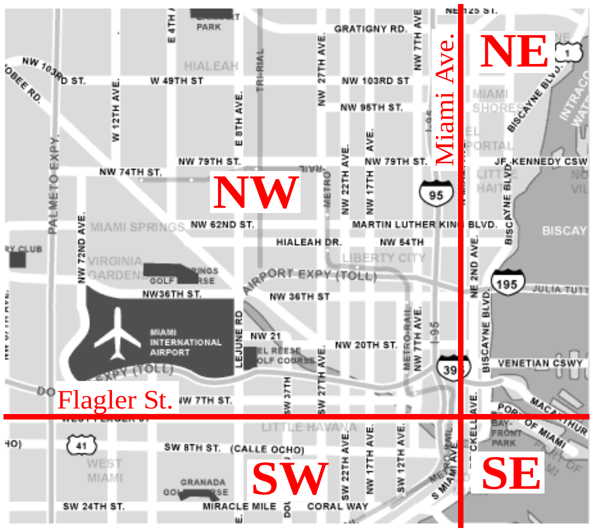
Chapter 3

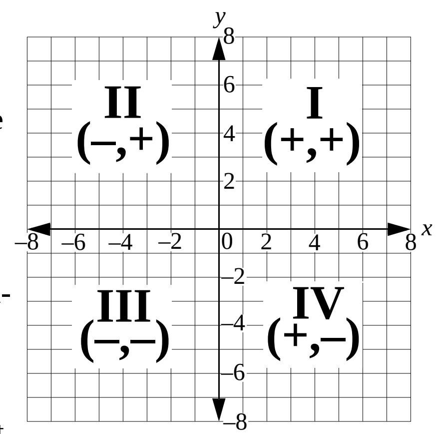
Linear Equations

Section 3.1
Coordinate Plane

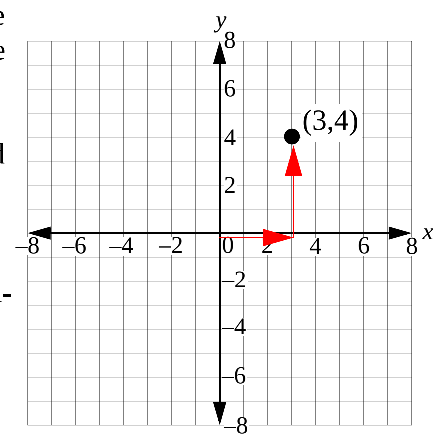


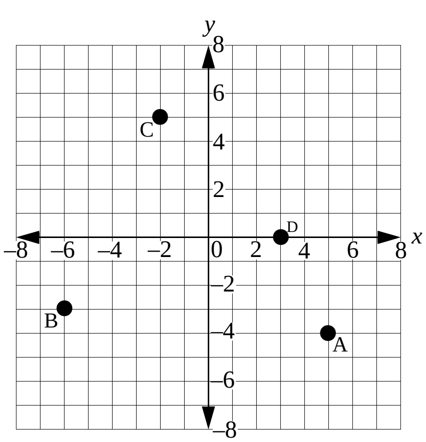
The map of the city of Miami divides the city into four sections or quadrants. Starting from the top right and moving in a counter-clock direction, the quadrants are Northeast (NE), Northwest (NW), Southwest (SW), and Southeast (SE). Notice how Flagler Street divides north from south and how Miami Avenue divides east from west. This arrangement of dividing streets and avenues is called a "coordinate plane."

In math, we use the coordinate plane to plot points. Points plotted on a coordinate plane are called a "set of coordinates" or a "coordinate pair." Just like in the map of the city of Miami, a coordinate plane is divided into four quadrants. Named Roman numerals I, II, III and IV, these quadrants divide both axes into positive and negative sides. Because in the coordinate plane *positive* is to the right of the *vertical* axis AND above the *horizontal* axis, quadrant I is positive-positive. Quadrant II is negative-positive (to the left of the *vertical* axis and above the *horizontal* axis). Quadrant III is negative-negative (to the left of the *vertical* axis and below the *horizontal* axis). Quadrant IV is positive-negative (to the right of the *vertical* axis and below the *horizontal* axis).



To make our work simpler, we have named the horizontal axis "*x*" and the vertical axis "*y*." Also, because in the alphabet *x* is before *y*, the coordinate sets will always be written in the order (*x*,*y*). Therefore, when, for example, in algebra we write point (3,4), this means to go 3 in the *x* direction and 4 in the *y* direction. It falls into the first quadrant. See coordinate plane to the right.





Examples

Write a set of coordinates for the points shown in the graph to the left.

1. Point A is set 5 units to the right and –4 units down. Its location is in the IV quadrant. (5,–4)
2. Point B is –6 units on the *x*-axis and –3 unit along the *y*-axis, placing it in the III quadrant. (–6,–3)
3. Point C is –2 units away from the origin along the *x*-axis and 5 units away along the *y*-axis. (–2,5) is in the II quadrant.
4. Point D is ON the *x* axis 3 units away from the origin, with no value for *y*, and no specific quadrant. (3,0).

Practice

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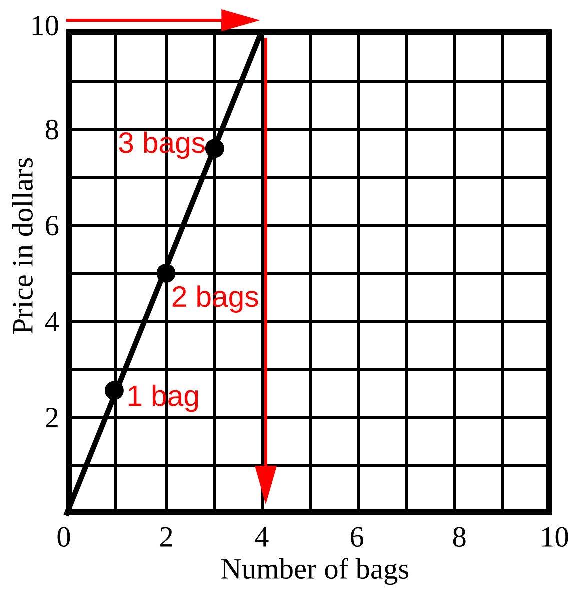
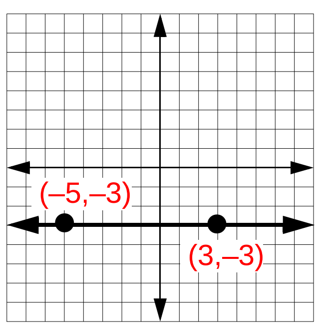
Section 3.2
From Linear Pattern to Equation

A person goes to the supermarket and reads that the price of a bag of sugar is \$2.50. This means the price of two bags is \$5.00 (2 times 2.50) and three bags \$7.50 (3 times 2.50). Or perhaps he reads that three apples go for \$0.25, and six apples, \$0.50, and so on. These examples are what is called a "linear pattern." Linear patterns, when plotted, form a line. The "bag of sugar" linear example (see graph) can be plotted to show this.

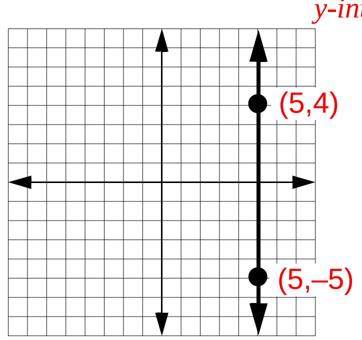


Necessary information to plot a line:
A line is made up of points and at least two points are needed to plot a line. Because lines must be drawn on a surface (two dimensions), each point's location needs two numbers. Called coordinates, these numbers are represented by letters (*x* and *y* are the most commonly used). Therefore, to plot a line, four numbers are needed: an *x* and a *y* for point one, and an *x* and a *y* for point two. The example, the graph matches 1 bag (*x* value) and \$2.50 (*y* value), 2 bags and \$5, 3 and 7.5. How many bags will you get for \$10? Answer: four bags.

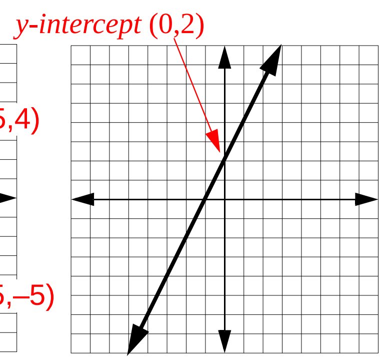
How to turn a line into an equation:
Straight lines on a flat surface can be drawn *horizontally, vertically* and *diagonally.* When plotted on a graph, the *y* values of all points on a *horizontal* line are always the same (left graph); conversely, the *x*



horizontal, $y = -3$



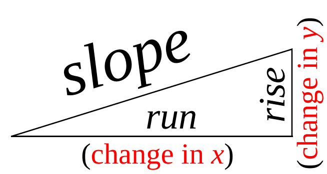
vertical, $x = 4$



diagonal, $y = 2x + 2$

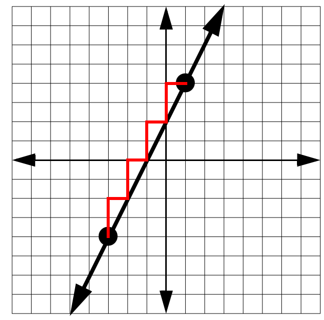
values of all points on a *vertical* line are always the same (center graph). Each point of a *diagonal* line has different values for *x* and *y* (right graph). To define—or locate—a straight line on a flat surface, it is best to show where the line crosses the vertical axis (*y* axis) and what the *slope* of the line is. The *y-intercept* (right graph) tells us where to find the first point of the line, and the slope shows the direction the line takes after crossing the *y* axis.

How to find the slope: A slope is like a ramp, where the rate of climb is determined by comparing the vertical distance *(rise)* over the hor-



$$slope = \frac{change\ in\ y}{change\ in\ x}$$

izontal distance (*run*). In the case of a graph, it means that the slope is a ratio (a fraction) of the change of y (vertical) over a particular change of x (horizontal).



For example, the diagonal line shown to the right goes through points (–3, –4) and (1, 4) and has a slope of 2. The slope is 2 because for every time y climbs (changes) two units, x moves to the right once $\left[\frac{y}{x} = \frac{2}{1}\right]$. At the same time, the y-intercept is where the line crosses the y-axis and in this particular case is 2. Therefore, the equation is written as:

$$y = 2x + 2$$
$$y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{y\text{-intercept}}$$

where m, the slope, is always the coefficient (number in front) of x, and b is the y-intercept (the point where the line crosses the y axis).

Example:
Write the equation for the line whose slope is 3 and y-intercept 2.

Slope = 3 = m $\quad$ y-intercept = 2 = b $\quad$ Answer: $y = 3x + 2$

Example:
Write the equation for the line whose slope is $-\frac{3}{2}$ and y-intercept 5.

$m = -\frac{3}{2}$ $\quad$ y-intercept = 5 $\quad$ Answer: $y = -\frac{3}{2}x + 5$

Example:
Write the equation for the line whose slope is –1 and y-intercept –3.

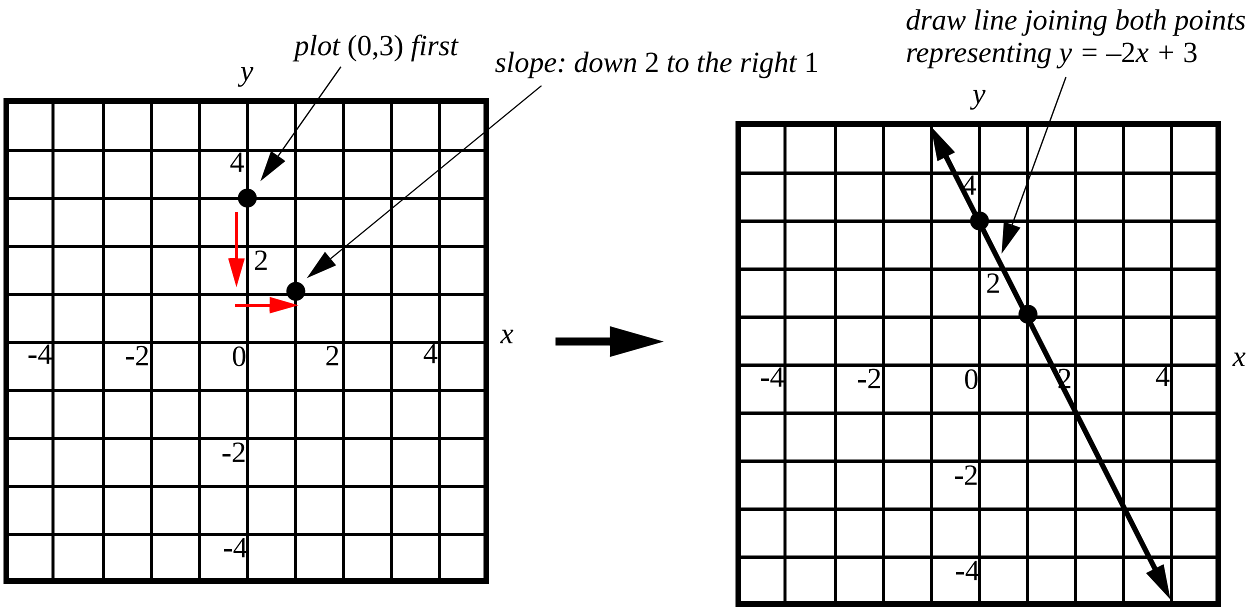
$m = -1$ $\quad$ y-intercept = –3 $\quad$ Answer: $y = -x - 3$

DOING IT BACKWARDS (plotting the line of an equation)

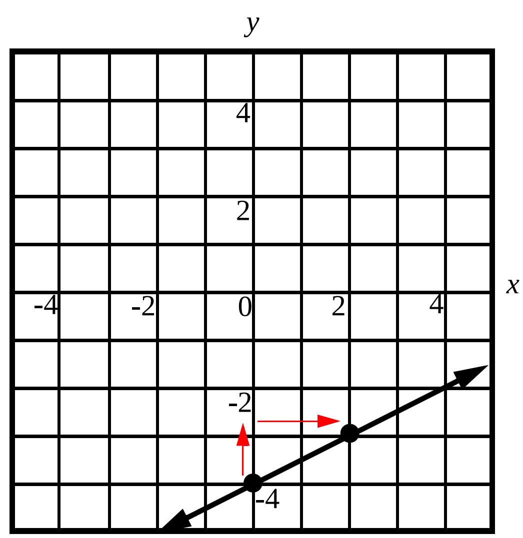
Example:
On a graph, plot line $y = -2x + 3$ (see top two graphs next page)

First plot the y-intercept value +3. Because the slope, $-\frac{2}{1}$, is the ratio of the *change in y* (–2) over *the change in x* (1), then this slope means that whenever the value of y moves down (negative) twice, the value of x will move once to the right (The slope is negative when only one of the variables is negative.). So the second point of the line is found by moving down to point (0,1) on the y axis (two spaces) and then one to the right to point (1,1). Finally, draw the line representing

$$y = -2x + 3.$$



Example: Plot line $y = \frac{1}{2}x - 4$



Plot y-intercept value at –4.

Second point: Using the slope, $\frac{1}{2}$, UP one (y), to the RIGHT 2 (x). Draw line that passes through the two points.

Remember:
When the slope is positive, move up ($+y$) and to the right ($+x$), and when is negative move down ($-y$) and to the right ($+x$). The x (horizontal) move is always to the right.

Practice
Write the linear equation that represents each of the lines shown.

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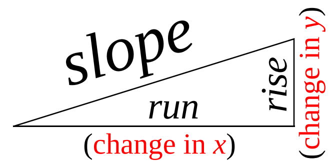
Section 3.3
The Slope

In section 3.2, linear equations and the use of the slope were introduced. Now the question will be how to come up with the slope and its corresponding linear equation without the use of graphs. In other words, analytically: using symbols and numbers only.

The diagram shows a ramp where the slope is defined as the constant ratio (if the slope is constant the line will always be straight) of the change in *y* over the change in *x*, or:



$$slope = m = \frac{change\ of\ y}{change\ of\ x}$$

If we try to find a *change* for *y*, we need to know where *y* begins and where it ends, and, of course, the same must be done for *x*: We need to know where *x* begins and where it ends.
To do this we use the coordinate values of two points on the line, then subtract the coordinates' values of *y* and the coordinates' values of *x*. The equation to find the slope is:

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

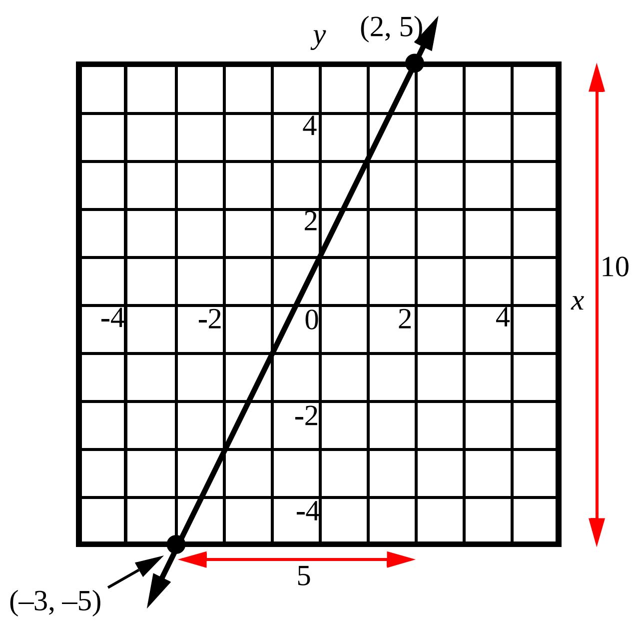
x_1 y_1 x_2 y_2

Example: The line beginning or passing through point (2, 5) and ending or passing through point (–3, –5) has the slope

$$\frac{-5-5}{-3-2} = \frac{-10}{-5} = 2$$

It doesn't matter which point is labeled 1 or which point labeled 2, as long as we don't mix the coordinates. Doing it backwards you get the same result:

$$\frac{5-(-5)}{2-(-3)} = \frac{10}{5} = 2$$



In the graph to the right, notice that the vertical (up and down) difference between the two points is 10: This is the difference of the values of ***y***. And the horizontal distance between the two points is 5: This is the difference of the values of ***x***. If the *y* distance is 10, and the *x* distance is 5:

$$slope = m = \frac{change\ of\ y}{change\ of\ x} = \frac{10}{5} = 2$$

It agrees with the above work that the slope is 2.

Example: Find the slope of the line passing through points (5, –4) and (7, 8).

If $y_2 = 8$ and $y_1 = -4$ then $8 - (-4)$
If $x_2 = 7$ and $x_1 = 5$ then $7 - 5$

Write it as the slope equation using ***m*** for the slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Then: $m = \dfrac{8-(-4)}{7-5} = \dfrac{12}{2} = 6$

Example: Find the slope of the line passing through points (–5, 6) and (2, –8).

$$m = \frac{-8-6}{2-(-5)} = \frac{-14}{7} = -2$$

FINDING THE EQUATION OF A LINE WHEN THE SLOPE AND A POINT ARE KNOWN

Because the slope is part of an equation, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, we can substitute the slope and one of the points in the equation to find the ***y-intercept*** equation (see section 3.2). In the example above ***m* = –2** and using the second point **(2, –8)**, the slope equation becomes:

$-2 = \dfrac{y-(-8)}{x-2}$ Solving the equation for *y*: $\dfrac{-2}{1} = \dfrac{y+8}{x-2}$

Multiplying across: $y + 8 = -2(x - 2)$

Distributing the right side: $y + 8 = -2x + 4$

Subtracting 8 from both sides to isolate ***y***: $y + 8 - 8 = -2x + 4 - 8$

$y = -2x - 4$

Where the *slope*, *m,* is –2 and the *y-intercept* is –4.

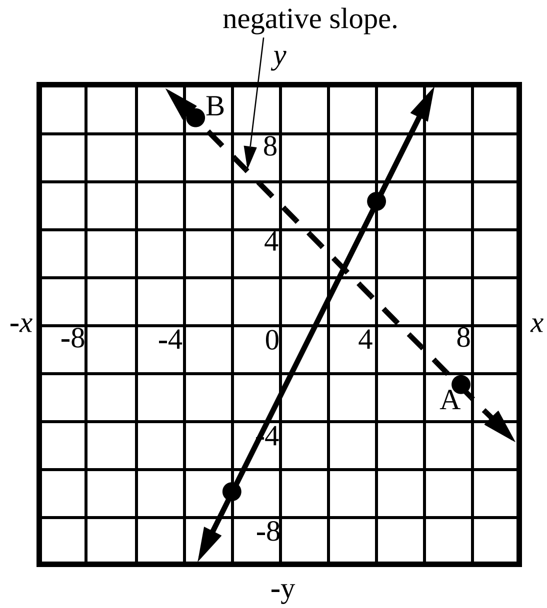
SLOPE, LINES, TABLES AND EQUATIONS

The idea that a linear relationship can be expressed as a line is based on the fact that every line has a constant slope (it has to go somewhere) and points (coordinates) on a line can be paired on a table. From there, it follows that a mathematical relationship (equation) may be set up to show that for every value of ***x***, there is one and only one corresponding value for ***y*** (because it is a straight line).

Example:

The points (4,5) and (–2,–7) form a line. Plot the points, write the *y*-intercept equation for the line, and find three other points on the line.

There are various way to approach the solution. If we are good at drawing, we can try for a graphical solution.

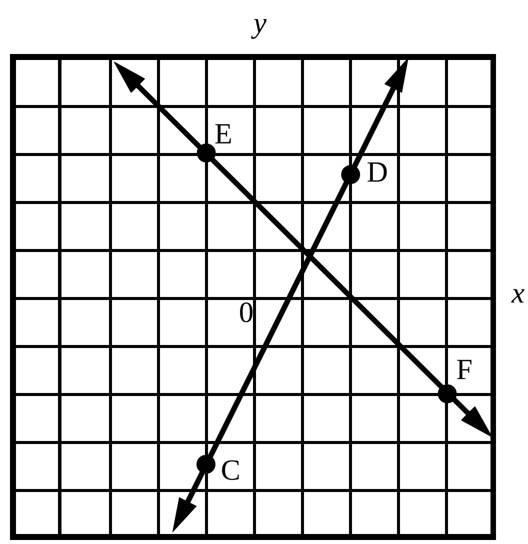


In the graphical solution the points are plotted and the line drawn. From the line, we select the value of *y* when *x* is zero (*y*-intercept). In this case the line crosses the *y*-axis at –3, so the *y*-intercept is –3. Next, find the slope. Because the slope is $\frac{y}{x}$, and the *y* distance between the two points is 12 and the *x* distance between the two points is 6, then the slope is $\frac{12}{6} = 2$. Now determine if the slope is positive or negative. We do this by looking at the line. **If the line slopes UP from LEFT TO RIGHT, the slope is positive, like in this case, but if the line slopes UP from RIGHT TO LEFT, the slope is negative**. The dashed line on the graph shows this.

Having determined that the *slope* is 2 and the *y*-intercept is –3, we are ready to write the equation for the line:

$$y = 2x - 3$$

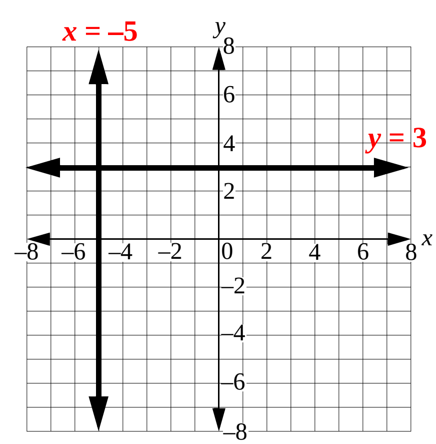
slope   *y*-intercept



Another way to determine the slope: (See second graph.) Moving from point C to point D (up and to the right), the *x* and *y changes* are +, +, which means that the slope is positive.

Moving backwards from point D to point C, the *x* and *y changes* are –,–. Therefore, the slope shows again that it is positive. If you try the same approach with line $\overline{EF}$, the *x* and *y changes* from E to F will be +,– and from F to E –,+, thus, in both instances, the slope will be negative.

There are two situations where the value of the slope is easily identifiable: When the line is horizontal and when the line stands up completely vertical. When the line is completely flat there is no value for *y* (no **rise**) and, therefore, the slope is *zero*; when the line is vertical there is no value for *x* (no **run**) and, therefore, because the slope is so large, it cannot be defined. Because of this, equations for horizontal lines contain no *x* and the equations for vertical lines contain no *y*. For example, *x* = –5 means that this is a vertical line that comes down through –5. On the other hand, *y* = 3 is a horizontal line that goes through 3. In the last graph, you can see both examples.



To relate all of this to three different values—or points—for the same line, build a table of corresponding values. Notice that the given point (4,5) is there (in red); the other points were found by the use of the slope $\frac{6}{3} = \frac{2}{1}$, which says that the value of y will change 6 times for every time x changes 3 times, or by reducing the ratio to 2 over 1, the value of y will change 2 times for every time that x changes once. Therefore, notice how the value of x goes 2, 3, 4, 5 (changes by 1) and the value of y goes 1, 3, 5, 7 (changes by 2). Why? Because the slope is 2.

x	y
2	1
3	3
4	**5**
5	7

Example: Plot line $3x + 4y = 12$.

There are two ways to answer this problem: By building a table and graph, or by writing the y-intercept form.

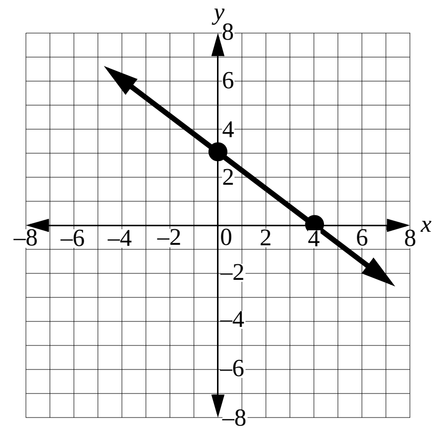
BUILDING A TABLE

Using a table we find two points. The most convenient ones are the y- and x-intercepts (when $x = 0$, and when $y = 0$). Although plotting more points increases line accuracy when you build a table, two points are sufficient to just draw a line, and the y- and x-intercepts happen to be the easiest to get.

When $x = 0$
$$3(0) + 4y = 12$$
$$4y = 12$$
$$y = 3$$

When $y = 0$
$$3x + 4(0) = 12$$
$$3x = 12$$
$$x = 4$$

x	y
0	**3**
1	2.25
2	1.5
3	0.75
4	**0**

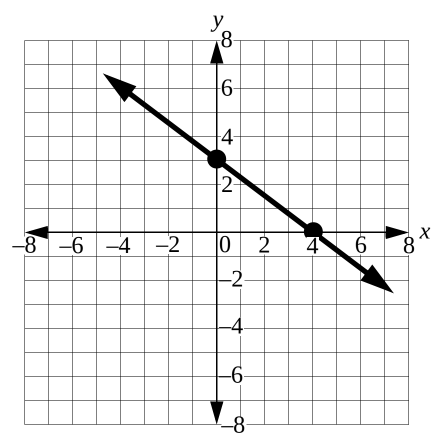


Using the y-intercept:

$$3x + 4y = 12$$
$$4y = -3x + 12 \quad (-3x \text{ on both sides})$$
$$y = \frac{-3x}{4} + 3 \quad (\text{divide by 4 both sides})$$

slope $\quad$ y-intercept

The *slope-intercept* equation shows that the y-intercept is 3 and the slope is $\frac{-3}{4}$. Plot the y-intercept (what's the value of y when x is zero) first by going to the origin (0,0) and moving up +3 along the y-axis. Then, using the slope, we plot the second point by starting from (0,3), go down 3 (the y value of the slope is negative, up if the slope were positive) and to the right (the x value of the slope) 4.



Notice that both graphs are identical, yet plotted differently.

Example: Is point (–7,3) a solution to line $4x + y = 18$?

To say that a point is a solution is the same as saying the point is ON the line. And to find out if the point is ON the line, the coordinates of the points, in this particular case $x = -7$ and $y = 3$, must make the equation true: *Left side = Right side*.

$4(-7) + 1(3) = -25$ (NOT a solution)

x y

Trying another point, (6,–6)

$4(6) + 1(-6) = 18$ (It is a solution)

Example: Write the equation in *standard* form for a line that crosses point (3,–5) and has slope $-\frac{1}{3}$.

Using the slope equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, if the slope is $-\frac{1}{3}$ and the point $(\overset{x}{3},\overset{y}{-5})$ then: $-\dfrac{1}{3} = \dfrac{y-(-5)}{x-3}\begin{matrix}\text{point } y\\ \text{point } x\end{matrix}$

Multiplying across: $3(y + 5) = -1(x - 3)$

Distributing: $3y + 15 = -x + 3$

Subtracting 15 on both sides: $3y + 15 - 15 = -x + 3 - 15$

$3y = -x - 12$

Writing it in standard form: $x + 3y = -12$

"Standard" form means Ax + By = C, where both "x" and "y" are to the left of the equal sign. Compare this to the "slope-intercept" form, where "y" and "x" are to either side of the equal sign ($y = mx + b$), and "general" form, where all three values are equal to zero (Ax + By + C = 0).

Practice:

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Section 3.4
Parallel and Intersecting Slopes

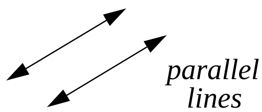

Figure 1

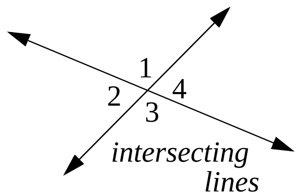

Figure 2

The multiple lines we draw on a plane (or a piece of paper) may cross or not cross at all. If they do NOT cross, lines are said to be parallel; if they cross, they will form four angles.

When the four angles are equal, they measure 90º each. Lines that cross at 90º are said to be perpendicular (⊥).

The way to tell if two equations represent lines that are parallel or perpendicular is by the slope: If the slopes of two or more lines are equal, the lines must be parallel (Figure 1) and the distance between any two lines is always the same. If, on the other hand, the slopes are not equal, it means the lines cross (Figure 2). If the lines cross at 90º, then the slopes are *reciprocal* AND *opposites* of each other (Figure 3).

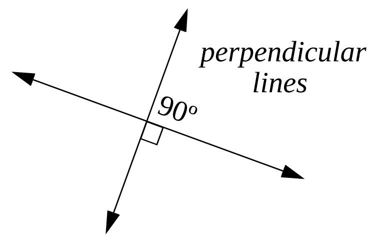

Figure 3

A *reciprocal* is a rational number that is written upside-down:

the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ the reciprocal of $\frac{3}{1}$ is $\frac{1}{3}$ the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or 2

The *opposite* of a number is the number with the "sign" changed:

the opposite of 4 is –4 the opposite of $-\frac{1}{5}$ is $\frac{1}{5}$

Therefore,

the *reciprocal* AND *opposite* of $\frac{1}{4}$ is $-\frac{4}{1}$ or -4

the reciprocal opposite of $-\frac{5}{4}$ is $\frac{4}{5}$

Because $\frac{1}{1} = 1$, the reciprocal opposite of 1 is –1

Example:
Are lines represented by equations $y = 2x - 7$ and $x + 2y = 5$ parallel, perpendicular, or neither of the two?

By looking at the first equation, we can identify the slope as the coefficient of x. Therefore, the slope of the first equation, $y = 2x - 7$, is 2.

Now we write the second equation in y-intercept form by subtracting x from both sides.

$$x + 2y = 5$$
$$x - x + 2y = 5 - x$$
$$2y = -x + 5$$

Dividing both sides by 2 $\quad \frac{2}{2}y = -\frac{x}{2} + \frac{5}{2}$

yields the *y-intercept* equation. $\quad y = -\frac{1}{2}x + \frac{5}{2}$

The slope of the second line is $-\frac{1}{2}$ (coefficient of x).

Because the slopes, 2 and $-\frac{1}{2}$, are *reciprocal* AND *opposites*, the lines are perpendicular.

Example:
Are the lines represented by equations $\quad 3x - 2y = 14 \quad$ and $\quad 2x - 3y = -15$
parallel, perpendicular, or neither of the two?

Turn first equation into y-intercept.

$$3x - 2y = 14$$
$$3x - 3x - 2y = -3x + 14$$
$$-2y = -3x + 14$$
$$y = \frac{-3}{-2}x - \frac{14}{2}$$
$$y = \frac{3}{2}x - 7$$

The slope is $\frac{3}{2}$ (coefficient of x).

Turn second equation into *y-intercept.*

$$2x - 3y = -15$$
$$2x - 2x - 3y = -2x - 15$$
$$-3y = -2x - 15$$
$$y = \frac{-2}{-3}x - \frac{15}{-3}$$
$$y = \frac{2}{3}x + 5$$

The slope is $\frac{2}{3}$ (coefficient of x).

Because the slopes, $\frac{3}{2}$ and $\frac{2}{3}$, are neither *equal* nor *reciprocal opposites*, the lines cross, but are not perpendicular.

Example:
Are the lines represented by equations $\quad y = x - 6 \quad$ and $\quad 3x - 3y = 11$
parallel, perpendicular, or neither of the two?

The slope of the first equation, taken from the coefficient of x, is 1.

Turn second equation into y-intercept form.

$$3x - 3y = 11$$
$$3x - 3x - 3y = -3x + 11$$
$$-3y = -3x + 11$$
$$y = -\frac{3}{-3}x + \frac{11}{-3}$$
$$y = x - \frac{11}{3}$$

The slope is 1 (coefficient of x). Because the slopes, 1 and 1, are equal, the lines are parallel.

Practice:

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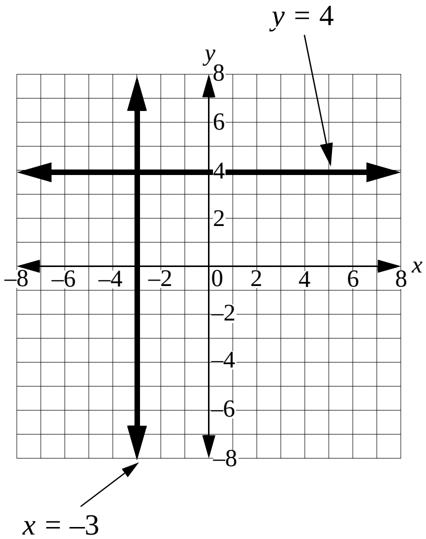
Section 3.5
From Equation to Line, from Line to Equation

Equation 1 on the right represents a horizontal line. It is a horizontal line because the slope (the coefficient of x) is zero. We know it is zero because to plot a line we always start with the slope-intercept equation $y = mx + b$ and the "mx" portion of the equation gets eliminated when $m = 0$, leaving only the value of b (the y-intercept) in the equation [$y = (0)m + 4$].

$$y = 4 \qquad (1)$$

$$x = -3 \qquad (2)$$

$$y = -2x + 1 \qquad (3)$$



If $y = (0)x + 4$ then $y = 4$

Equation 2 above is a vertical line. For vertical lines, the slope is undefined (infinite) because the *change* of x is zero. NOTICE EQUATIONS HAVING ONLY ONE VARIABLE ARE EITHER HORIZONTAL OR VERTICAL.

Equation 3 represents a diagonal line in the *slope-intercept* form.

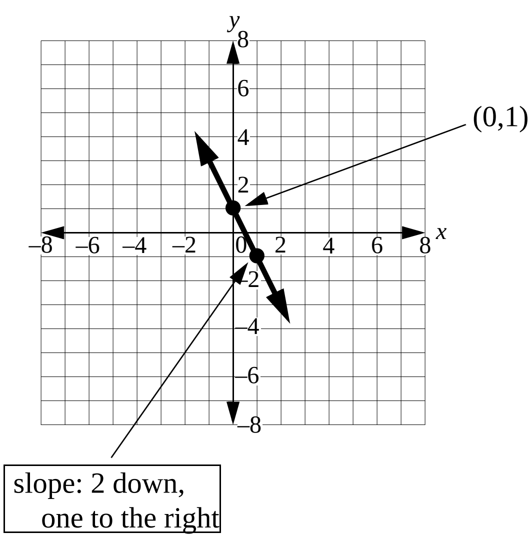
Plotting a line using the slope-intercept form

The *slope-intercept* form, as its name indicates, provides us with the y-intercept and *slope* values by just looking at it.

$$y = mx + b$$

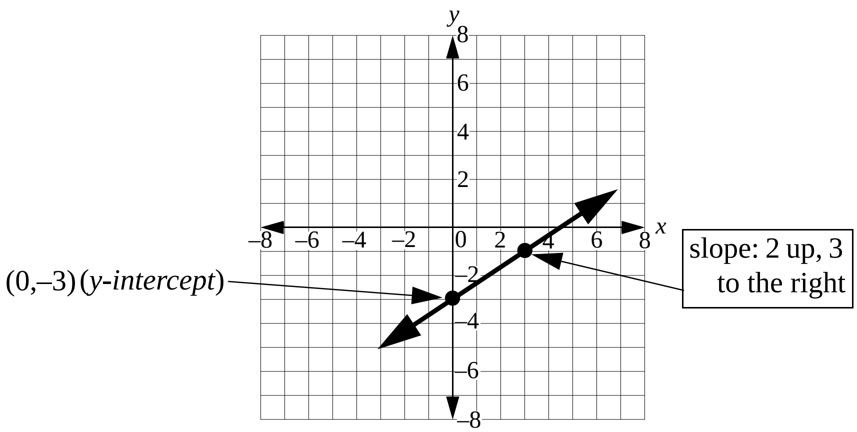
Where m is the slope and b is the y-intercept. For equation 3 above, the y-intercept value is 1 and the *slope* is –2.

We plot the *y-intercept* (the place where the line will cross the *y-axis*) FIRST—in this case (0,1) because, for the y-intercept, the value of x is always zero. Secondly, we find the next point by moving two spaces down and one to the right because the slope is $m = \dfrac{change\ \ in\ \ y}{change\ \ in\ \ x}$ or $\dfrac{-2}{1}$ and y, being negative, moves down and x, being positive, moves to the right. Only two points are needed to form a line. Join both points and you have drawn the line that defines the equation. See graph to the right.



Example: Plot equation $y = \dfrac{2}{3}x - 3$

First plot y-intercept at –3, then, starting *at the y-intercept*, plot slope 2 up (y is positive) and 3 to the right (x). (See plot on next page).



Example: Plot equation $2x - 5y = 10$

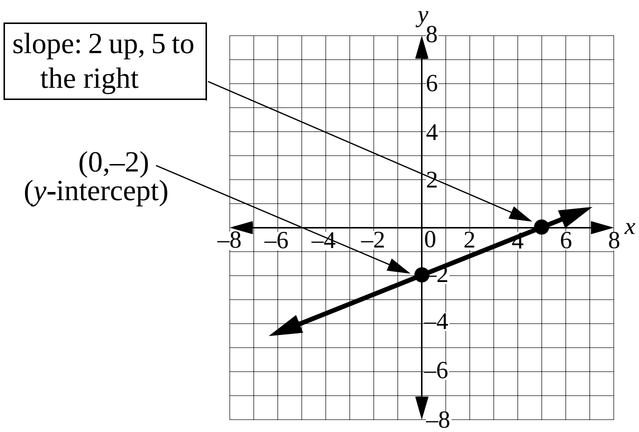
First express equation in *y*-intercept form:

$$-5y = -2x + 10$$

$$-\frac{5}{-5}y = -\frac{2}{-5}x + \frac{10}{-5}$$

$$y = \frac{2}{5}x - 2$$

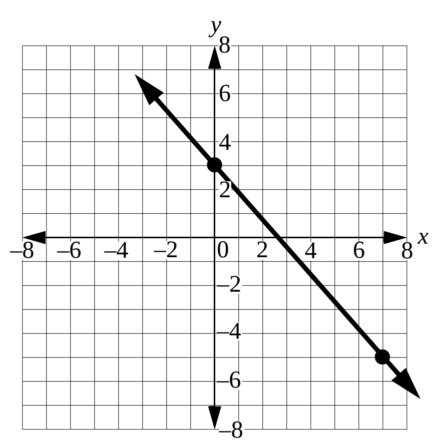
y-intercept = –2 slope = $\frac{2}{5}$



From a graph, find the equation of the line.

In the graph to the right, the line crosses the *y-axis* at 3 and the *slope* (the *y* change between the two points is negative—8 down—over the *x* change which is 7) is $-\frac{8}{7}$.

Using these numbers in the slope-intercept form ($y = mx + b$), $m = -\frac{8}{7}$ and $b = 3$. Therefore, $y = -\frac{8}{7}x + 3$.



Practice:

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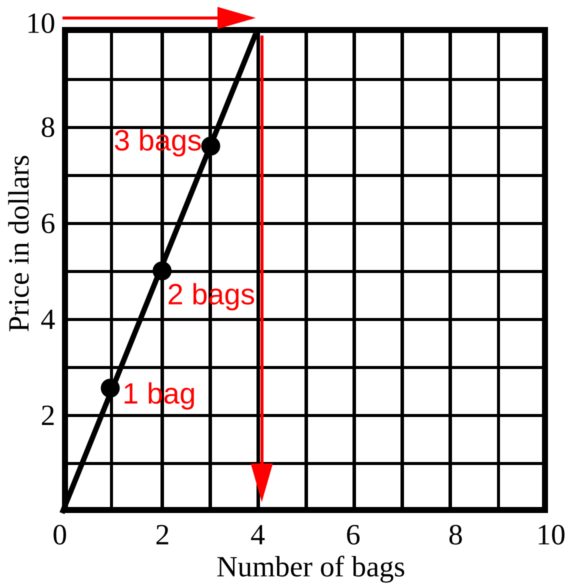
Section 3.6
Linear Models

By now you should have a basic idea of what “linear” in mathematics means: A way of connecting the relationship between two variables, where the relationship in question may be represented by a straight line. Therefore, the “linear” equations we have been discussing in the previous sections of chapter three, help us solve “linear problems.”

At the beginning of section 3.2, the graph shown below is given as an example representing a relationship between two variables, in this case bags and price.



What is a model?

In mathematics, a model is a reproduction or replica that is capable of projecting on what we already know. In other words, mathematical models use historical data to predict different make-believe outcomes to help us make decisions. In this section, only linear models will be discussed.

An equation describes the situation.

Because it helps us in predicting how much any number of bags would cost, the graph to the left is a linear model. Using what we already know, the slope of the line is:

$m = \frac{5}{2}$ and the y-intercept is zero

$\boldsymbol{y = mx + b}$

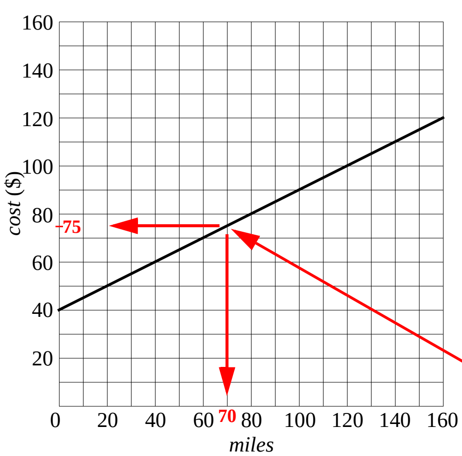
$\boldsymbol{b = 0}$

The equation of the line is $y = \frac{5}{2}x$

We can now compute the price of any number of bags by the use of the **linear model** represented by the equation. X represents the number of bags and y the price. Or we can change the variable x for b (bags) and y for p (price) to make it $p = \frac{5}{2}b$. For example, if b is 150 then p is: $p = \frac{5 \times 150}{2} = \$375$

Example:

To move, Pat needs to rent a truck for a few hours. Build a linear model to predict her costs using the following specifications: It costs \$40 per day and \$0.50 per mile to rent the truck she wants.



We begin with the condition that Pat must pay \$40 for starting the truck (zero miles). Therefore, the line for this model starts at 40. From 40 the line climbs at the rate of 50 cents per mile or \$50 per hundred miles. The graph shows this.

The slope, m, is 0.5 (50/100) and the y-intercept is 40 ($miles = 0$)

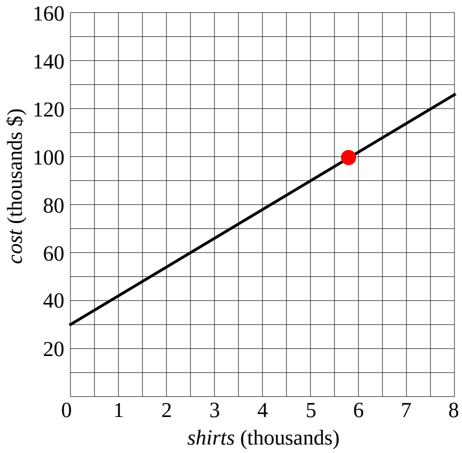
(*m = miles*)

The equation is $Cost = 0.5m + 40$ or $y = 0.5x + 40$

If she drives 70 miles: $y = 0.5(70) + 40 = \$75$

Example:

Ernestine wants to start a business making shirts. The machinery, and other equipment to do the manufacturing of the shirts, costs $30,000. She computed the material and labor costs of making one shirt and she arrived at a figure of $12. Write the equation that would describe Ernestine's linear model.



Before the first shirt is made, she pays $30,000. Thus the first shirt will cost $30,012, the second shirt $30,024, an so on. Thus the slope of the line is the vertical change (y = 12) over the horizontal change (x = 1):

$m = \dfrac{12}{1} = 12$ and the y-intercept (when $x = 0$) is 30,000

Ernestine's linear model is: $y = 12x + 30{,}000$

5,580 shirts will cost (red dot in graph) $y = 12(5580) + 30000 = \$96{,}960$

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