# Chapter 2

# Introduction to Equations

# Section 2.1
# Equations: Addition and Subtraction

An equation is an open statement that has one or more unknowns, also called variables.

**Example**:

$x + 5 = 12$ or $y - 15 = 8$ ($x$ and $y$ are variables)

Equations are solved by the use of *inverse* rules (see page 16) and doing the opposite. The value of the unknown is found because inverse rules undo what the equation states. A typical inverse rule says additions are undone by subtractions and subtractions with additions. Therefore, the first equation above is:

$$x + 5 - 5 = 12 - 5 \quad \text{(addition undone by subtraction)}$$
$$x + 0 = 7$$
$$x = 7$$

Notice that by undoing the +5 on the left with a –5, we are forced to add –5 on the right side of the equation to keep the equation balanced (equal). The value of $x$ is then exposed as $12 - 5 = 7$.

In the second equation above, –15 is being subtracted from an unknown "$y$."

$$y - 15 + 15 = 8 + 15 \quad \text{(subtraction undone by addition)}$$
$$y - 0 = 8 + 15$$
$$y = 23$$

Notice that by undoing –15 on the left with a +15, we are forced to add +15 on the right side of the equation to keep the equation balanced (equal). The value of $y$ is then exposed as $8 + 15 = 23$.

**Example**:

$$x - 6 = 10$$
$$x - 6 + 6 = 10 + 6$$
$$x + 0 = 16$$
$$x = 16$$

**Example**:

$$2x + 3 = x - 12$$
$$2x - x + 3 - 3 = x - x - 12 - 3$$
$$x + 0 = 0 - 12 - 3$$
$$x = -15$$

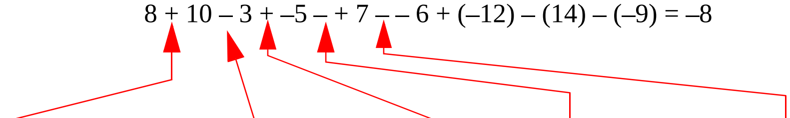
**Example**:

$$13 = a + 10$$
$$13 - 10 = a + 10 - 10$$
$$3 = a + 0$$
$$3 = a$$
$$a = 3$$

### "Sign sign everywhere a sign.... do this, don't do that, can't you read the sign?"*

Understanding how to handle positive and negative signs is hard for students entering algebra for the first time. Hopefully, the following examples and brief explanation will help.

$$8 + 10 - 3 + -5 - + 7 - - 6 + (-12) - (14) - (-9) = -8$$



In this example, 8 is added to 10 and then 3 is subtracted, –5 is added, 7 is subtracted, and –6 is subtracted... This shows we can *add negative* numbers (+ –5), *subtract positive* numbers (– + 7), *subtract negative* numbers (– – 6), and so on. Sounds confusing? Let's break it down using the number line (see page 21):

1. Start by adding 8 and 10 to get 18, and taking 3 away is 15. (So far, so good!).
2. Next, "add negative 5" (+ –5) to 15 and go down to 10 (adding "bad" to "good" makes it go down).
3. Next, "subtract positive 7" (– + 7) and go down to 3 this time (taking "good" away makes it go down).
4. Next, "subtract negative 6" (– – 6) and go up to 9 (taking "bad" away makes it "good", and goes up).
5. At this point, having a parenthesis is the same as not having one. Ignore it and continue.
6. Next, "add negative 12" [+ (–12)] and go down beyond zero to –3 (the negative side of the number line).
7. Next, "subtract positive 14" [– (14)] and go down to –17 (taking "good" away makes it go down further).
8. To finish, "subtract negative 9" [– (–9)] and go up to –8 (taking "bad" away makes it go up).

(Notice the similarities between step 6 and step 2, step 7 and step 3 and step 8 and step 4.)

## Taking a short cut

$$-12 + 9 - (-14) - (7) + (-11) - (+8) + 5 - -16 + -2 = 4$$

*Continues in page 30.*

**Practice**:
Solve.

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# Section 2.2
# Equations: Multiplication and Division

An equation is an open statement that has one or more unknowns, also called variables.

**Example**: $4x = 12$ or $\frac{x}{3} = 8$

Equations are solved by the use of inverse rules (see page 16). The value of the unknown is found because inverse rules undo what the equation states. A typical inverse rule says multiplications are undone by divisions and divisions by multiplications. Therefore, the first equation above is:

$$\frac{4x}{4} = \frac{12}{4}$$ Where the multiplication $4x$ was undone by division

$$x = 3$$

Notice that by undoing (dividing) 4 on the left with another 4, we are forced to divide by 4 on the right side of the equation to keep the equation balanced (equal). The value of $x$ is then exposed as $\frac{12}{4} = 3$.

In the second equation above, the variable $x$ is being divided by 3, therefore, to undo it, both the left and right-hand of the equation must be multiplied by 3.

$$\frac{x}{3}(3) = 8(3)$$

$$x = 24$$ The result is that the 3s on the left cancel and the answer is 8(3) = 24

**Example**:

$$5x = 35$$
$$\frac{5x}{5} = \frac{35}{5}$$
$$x = 7$$

**Example**:

$$54 = -6y$$
$$\frac{54}{-6} = \frac{-6}{-6}y$$
$$-9 = y$$

**Example**:

$$-12a = -48$$
$$\frac{-12a}{-12} = \frac{-48}{-12}$$
$$a = 4$$

*From page 28*

1. Remove double signs and all parentheses, leaving only numbers and one sign between numbers.
   —If the signs are the same, turn both signs into one positive sign.
   —If the signs are different, turn both signs into one negative sign.
2. Find answer by moving right and left on the number line.

The example in page 27 becomes: $-12 + 9 + 14 - 7 - 11 - 8 + 5 + 16 - 2 = 4$

### Multiplication and Division

Multiplication is noted with an ×, dot $(2 \cdot 3)$, or parenthesis, like in 3(4) = 12. Division is noted with a slash (/), fraction line (–), or ÷.

1. $9 \times 8 = 72$ same signs, answer positive
2. $12 \cdot -3 = -36$ different signs, answer negative
3. $-4(-7) = 28$ same signs, answer positive
4. $(8)-5 = -40$ different signs, answer negative
5. $-12/4 = -3$ different signs, answer negative
6. $\frac{-2.4}{-8} = 0.3$ same signs, answer positive
7. $84 \div (-12) = -7$ different signs, answer negative
8. $\frac{52}{13} = 4$ same signs, answer positive

If you look carefully at the answers of the eight multiplication and division examples above, one conclusion seems clear: *Answers are positive when the signs of both numbers are the same, and negative when they are not.*

**Practice**:
Solve.

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# Section 2.3
# Equations: Two or More Steps

An equation is an open statement that has one or more unknowns, also called variables.

**Example**: $3x - 12 = 6$

Equations are solved by the use of inverse rules (see page 16). The value of the unknown is found because inverse rules undo what the equation says. A typical inverse rule says addition is undone with subtraction, subtraction with addition, multiplication with division, and division with multiplication.

The objective of "solving for *x*" is to have "*x*" isolated (alone) to the left of the equal sign. This way, when we finally solve for "*x*", the value of "*x*" is read: "*x* is equal to...". Isolating "*x*" involves removing (undoing) every value (number or letter) to the other side of the equation using the inverse rules. Removal is done one number at a time, starting with the most convenient number.

Therefore, because in the above equation 12 is being subtracted and 3 is being multiplied, the equation above is solved by adding 12 and dividing by 3, on both sides:

$3x - 12 + 12 = 6 + 12$ add 12 to both sides (it's more "convenient" to add 12 first)

$3x = 18$

$\frac{3x}{3} = \frac{18}{3}$ divide by 3 (because 3 is closer to "x", it's more convenient to do 3 last)

$x = 6$

Notice that by undoing (adding) 12 on the left with another 12, we are forced to add another 12 on the right side of the equation to keep the equation balanced (equal). After the addition, $3x = 18$ is then undone by dividing both sides by 3. The value of $x$ is then exposed as $\frac{18}{3} = 6$.

**Example**: $2x + 10 = 6$

$2x + 10 - 10 = 6 - 10$ subtract 10 (both sides)

$2x = -4$

$\frac{2x}{2} = -\frac{4}{2}$ divide both sides by 2

$x = -2$

**Example**: $24 = -9 - 3y$

$24 + 9 = -9 + 9 - 3y$ add 9 to both sides

$33 = -3y$

$\frac{33}{-3} = \frac{-3y}{-3}$ divide both sides by –3

$-11 = y$

$y = -11$

**Example**: $5x + 9 = 2x - 27$

$5x - 2x + 9 - 9 = 2x - 2x - 27 - 9$ subtract $2x$ and 9

$3x = -36$

$\frac{3x}{3} = -\frac{36}{3}$ divide by 3

$x = -12$

**Practice**

Solve.

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# Section 2.4
# Formulas

A formula is an equation where the variables have some sort of meaning. By solving the formula (equation), we can find the value of one variable, if we know the value of all the others.

**Example**: The formula for distance is $d = rt$, where ***d*** represents distance, ***r*** represents speed, and ***t*** represents time. Therefore, if we know the values of any two, the third value is found by solving the formula.

Distance from Miami to Orlando = 235 miles
Average speed to Orlando = 60 miles per hour

Solving formula for time: $\frac{d}{r} = \frac{rt}{r}$ $\qquad \frac{d}{r} = t$ $\qquad$ or $\qquad t = \frac{d}{r}$

Therefore: $\qquad t = \frac{235}{60} = 3.92$ hours

Notice that it is easier to solve the formula for the variable you want, then place the numbers accordingly.

**Example**: The formula for computing interest is $i = Prt$, where ***i*** represents interest, ***P*** the principal (amount of money), ***r*** the rate of interest, and ***t*** time that the money is held.

If a person borrows $5,000 for 12 months (1 year) at 0.5% per month compounded annually, then the interest will be:

$$i = (5{,}000)(0.005)(12) = \$300 \text{ per month}$$

Notice that the percent was written as its decimal equivalent by moving the decimal point twice to the left (see percent section 1.5). Also notice that the rate of interest (%) is stated in the same units (months) as the variable time ($t$), thus the amount paid is per month.

**Example**: The formula for computing the circumference of a circle is $C = d\pi$, where ***C*** is the circumference, ***d*** the diameter, and $\pi$ (*pi*) the constant 3.14. Find the diameter of the circle whose circumference is 20 ft.

Solving for $d$, $\qquad \frac{C}{\pi} = \frac{d\pi}{\pi}$ $\qquad \frac{C}{\pi} = d$ $\qquad$ or $\qquad d = \frac{C}{\pi}$

Substituting 20 for $C$ and 3.14 for $\pi$ we get

$$d = \frac{20}{3.14} \qquad d = 6.37 \text{ ft.}$$

**Practice**:
Solve.

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