Chapter 12

Transition to Geometry

Section 12.1
Measurements

CONVERSION TABLES

Conversion tables are the key to solving measurement problems.

Customary: Length

	Miles	Yards	Feet	Inches
Miles	1	1,760	5,280	63,360
Yards		1	3	36
Feet			1	12

Customary: Weight

	Tons	Pounds	Ounces
Tons	1	2,000	32,000
Pounds		1	16

Metric: Length

	Kilometers	Meters	Centimeters	Millimeters
Kilometers	1	1,000	10,000	100,000
Meters		1	100	1,000
Centimeters			1	10

Metric: Weight

	Metric Tons	Kilograms	Grams	Milligrams
Metric Tons	1	1,000	1,000,000	1,000,000,000
Kilograms		1	1000	1,000,000
Grams			1	1,000

Customary: Capacity

	Gallons	Quarts	Pints	Cups	Ounces	Tablespoons	Teaspoons
Gallons	1	4	8	16	128	256	768
Quarts		1	2	4	32	64	192
Pints			1	2	16	32	96
Cups				1	8	16	48
Ounces					1	2	6
Tablespoons						1	3

Metric: Capacity

	Kiloliters	Liters	Milliliters
Kiloliters	1	1,000	1,000,000
Liters		1	1000

DIMENSIONAL ANALYSIS

Dimensional analysis is the way to convert units by the use of ratios.

Example: Convert 78 inches to feet.

1. Write a ratio of the given unit over one

$$\frac{78 \, inches}{1}$$

2. Multiply ratio by another ratio with the given unit in the denominator (bottom) and the unit "to convert to" in the numerator.

$$\frac{foot}{inches}$$

3. Using the conversion tables decide which unit is larger and place a 1 with the larger unit, and the conversion factor with the smaller unit.

$$\frac{1 \, foot}{12 \, inches}$$

$$\frac{78 \; inches}{1} \times \frac{1 \, foot}{12 \; inches} = \frac{78}{12} = 6.5 \quad \text{feet}$$

4. Cancel the name of the given units, multiply and divide to get the answer.

Example: Convert 20 quarts to cups.

$$\frac{20 \; quarts}{1} \times \frac{4 \, cups}{1 \, quart} = \frac{80}{1} = 80 \; \text{cups}$$

Example: Convert 500 milligrams to grams.

$$\frac{500 \; milligrams}{1} \times \frac{1 \, gram}{1000 \; milligrams} = \frac{500}{1000} = 0.5 \; \text{grams}$$

CONVERTING COMPOUND UNITS

A compound unit has more than one unit. For example, speed, in *miles per hour*, is a compound unit because it contains *miles* and *hours*. Compound units may be also converted using dimensional analysis by converting one unit at a time.

Example: Convert $\frac{55 \, miles}{hour}$ to $\frac{feet}{second}$.

from miles to feet; from hours to seconds

$$\frac{55 \; miles}{hour} \times \frac{5280 \; feet}{1 \, mile} \times \frac{1 \, hour}{3600 \, seconds} = \frac{55 \times 5280 \times 1}{1 \times 3600} = 80.7 \frac{feet}{seconds}$$

CONVERTING FROM CUSTOMARY TO METRIC (OR METRIC TO CUSTOMARY) UNITS

To convert between the two measuring systems, use dimensional analysis and the *metric-to-customary* conversion table. Only one unit per standard of measurement is needed (length, weight or capacity).

Customary to metric convertion table

Length	1 inch = 2.54 centimeters
Weight	1 pound = 454 grams
Capacity	1 quart = 0.946 liters

Example: Convert 8,000 feet to meters.

Because the table above uses inches for length, convert 8,000 feet to inches first:

$$\frac{8000\ feet}{1} \times \frac{12\ inches}{1 foot} = \frac{96000}{1} = 96,000\ \text{inches}$$

Next, convert inches to centimeters:

$$\frac{96000\ inches}{1} \times \frac{2.54\ centimeters}{1 inch} = \frac{243840}{1} = 243840\ \text{centimeters}$$

Finally, convert centimeters to meters:

$$\frac{243840\ centimeters}{1} \times \frac{1 meter}{100\ centimeters} = \frac{243840}{100} = 2,438.4\ \text{meters}$$

TEMPERATURE: Converting from ºCelsius to ºFahrenheit

To measure temperature using the metric system, the scale is called Celsius (ºC), while the scale to measure temperature in customary units is call Fahrenheit (ºF).

To convert and build a table from ºC to ºF, the following equation is necessary:

$$F = \frac{9}{5}C + 32$$

Example:
Convert 212 ºF (H_2O boiling point) to ºC

$$212 = \frac{9}{5}C + 32$$ Subtract 32 from both sides

$$180 = \frac{9}{5}C$$ Multiply by 5 and divide by 9 both sides

$$\frac{180(5)}{9} = C = 100\ \text{ºC}$$

Example:
Convert 37 ºC (body temperature) to ºF

$$F = \frac{9}{5}(37) + 32$$ Multiply 9 × 37

$$F = \frac{333}{5} + 32$$ Divide by 5

$$F = 66.6 + 32 = 98.6\ \text{ºF}$$

Practice:

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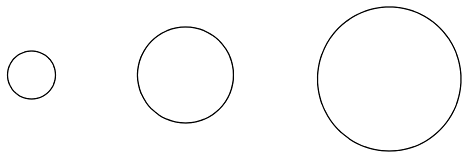
Section 12.2
Similarity

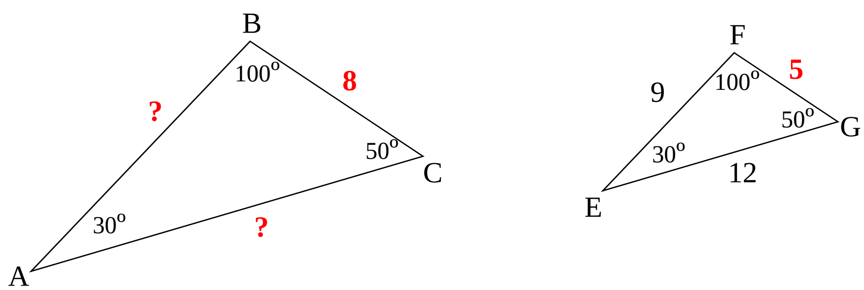
In geometry, similarity is a visual thing. Figures are similar if they are the same shape and have different measurement. In other words, they are either a smaller version or a larger version of the original. For example, by this definition, all three circles below are similar, but the measures of each one of them are different.

Examples:



Because sizes of similar shapes are proportional, *ratios* are used to solve similarity problems.

Example: In the two similar triangles shown below, find the two missing sides.



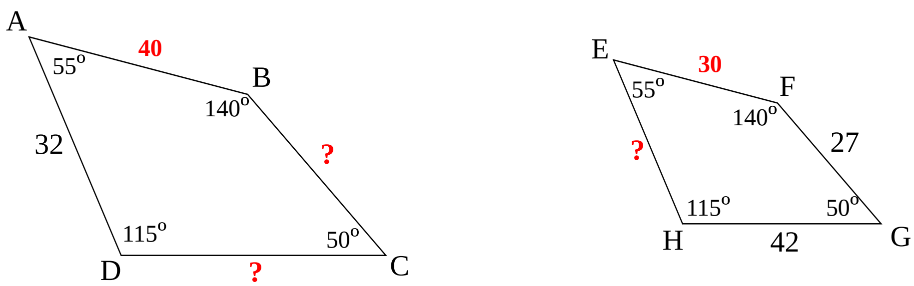
The corresponding, known sides are 8 and 5 (red bold), and two missing sides are $\overline{AB}$ and $\overline{AC}$. To find missing sides set up proportions using the corresponding sides of each triangle, then multiply numbers across and divide by the third number:

$$\frac{AB}{EF} = \frac{BC}{FG} \quad \text{and} \quad \frac{AC}{EG} = \frac{BC}{FG}$$

$$\frac{AB}{9} = \frac{8}{5} \quad \text{and} \quad \frac{AC}{12} = \frac{8}{5}$$

$$AB = \frac{9 \times 8}{5} = 14.4 \quad \text{and} \quad AC = \frac{12 \times 8}{5} = 19.2$$

Example: The two quadrilaterals shown below are similar. Find the missing sides.



Because the two quadrilaterals are similar, their corresponding sides are proportional.

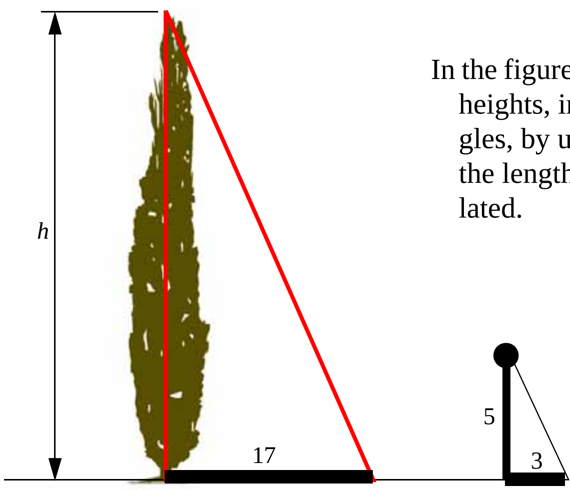
The corresponding known sides are 40 and 30 (in red), and the three missing sides are $\overline{BC}$ and $\overline{CD}$ and $\overline{EH}$. To find them set up the proportions:

$$\frac{40}{30} = \frac{BC}{27} \quad \rightarrow \quad BC = \frac{40 \times 27}{30} = 36$$

$$\frac{40}{30} = \frac{DC}{42} \quad \rightarrow \quad CD = \frac{40 \times 42}{30} = 56$$

$$\frac{40}{30} = \frac{32}{EH} \quad \rightarrow \quad EH = \frac{30 \times 32}{40} = 24$$

Example: Anybody can find the height of a tall tree without climbing to the top.



In the figure shown, the tree and the pole in the ground have different heights, in feet. Because their shadows make similar right triangles, by using the height and length of the shadow of the pole and the length of the shadow of the tree, the height of the tree is calculated.

$$\frac{height}{shadow} = \underset{\text{tree}}{\frac{h}{17}} = \underset{\text{pole}}{\frac{5}{3}}$$

$$h = \frac{5 \times 17}{3} = 28.\overline{3} \text{ feet}$$

Practice:

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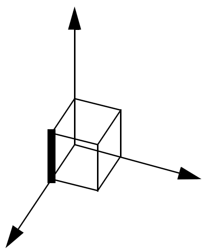
Section 12.3
Perimeter and Circumference

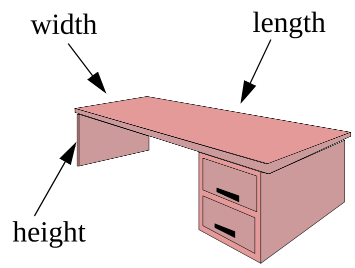


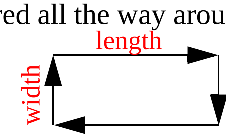
DIMENSIONS AND SPACE

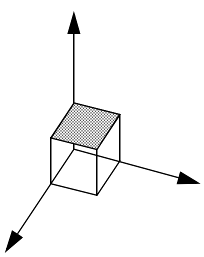
Every piece of matter has mass and takes space as a three-dimensional object, whether it is a person, rock or unobservable germ. Moreover, this mass can be measured in three different directions or axes. These axes are given many different names. For example, length, width, depth, height, thickness, altitude, and elevation. What they are called is a personal preference, but they will always be three.

Measuring dimensions

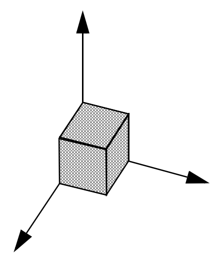
A *linear dimension* is measured in one direction only (see **bold** line in graph top-left). For example, *perimeter*. The perimeter of the desk shown is found by adding the desk's width and length twice. Twice because the top of the desk is a rectangle, has two widths and two length, and the perimeter must be measured all the way around.







Surface (two dimensions, see shaded portion in graph to the left) is the product (multiplication) of quantities measured in TWO directions: AREA. The area of the desk shown is the product of the length × width. Area measurements are given in *square units* (examples, ft^2 or m^2).



Space (three dimensions, see shaded portion in graph to the right) is the product of quantities measured in THREE directions: VOLUME. To find the volume of the box needed to pack the desk shown above, multiply the length × width × height of the desk. Volume measurements are given in cubic units (examples, ft^3 or m^3).

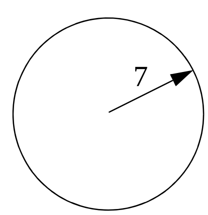
CIRCUMFERENCE

The curved perimeter (distance around) of circles is called *circumference*, and it is calculated by multiplying the constant $\pi \times diameter$ of the circle:

$C = d\pi$ OR $C = 2r\pi$ *Where:*

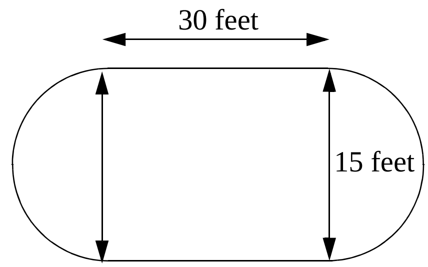
d = *diameter*
r = *radius*
π = *3.14*

Example: Find the perimeter of a circle with radius of 7 inches.



$C = 2(7)(3.14) = 43.96$ inches.

Example: Find the perimeter of the swimming pool shown.



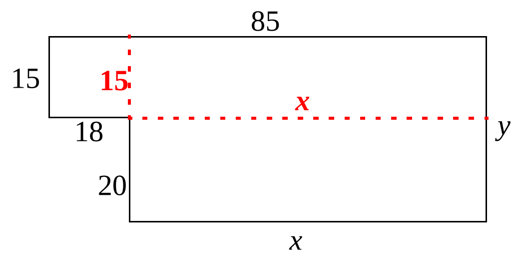
The swimming pool is formed by two (top and bottom) straight lines of 30 feet each, and two half-circles (left and right) with a diameter of 15 feet each. (Notice that two half-circles make one whole circle.)

$$\text{Total perimeter} = 30 + 30 + \frac{Circumference}{2} + \frac{Circumference}{2} = 60 + C$$

$$C = d\pi = (15)(3.14) = 27.1$$

$$\text{Total perimeter} = 60 + 27.1 = 87.1 \text{ feet}$$

Example: Find the perimeter, in feet, of the floor plan shown.



To find the perimeter, all the sides of the irregular shape must be added; however, neither length x nor length y are shown.

Because 18 and x are horizontal lengths which add up to the top measure of 85, then x can be found by subtracting:

$$85 - 18 = x = 67$$

The width, y, is also the sum of the width 15 and 20 shown to the left of the plan. Therefore: $y = 35$

and the perimeter of the floor plan is, clockwise: $85 + 35 + 67 + 20 + 18 + 15 = 240$

Practice:

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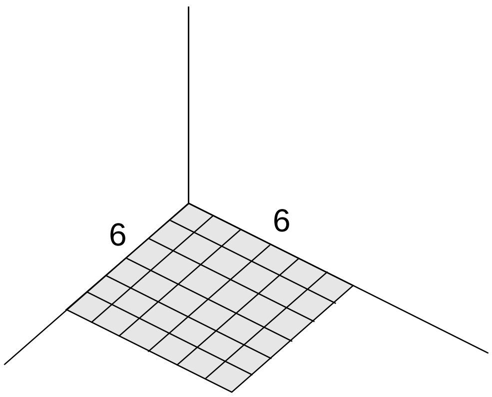
Section 12.4
Area

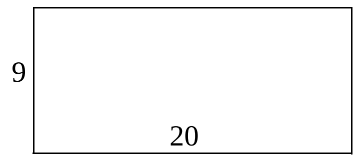
Area is the way *surface* is measured. Surface could be a floor, a yard, a wall, or the contact area of a tire against the road. Area is the product (multiplication) of quantities measured in TWO directions.



In the graph below, area is the plane that has been darkened and it accounts for every one of the smaller squares inside the area.

The graph has 36 square units, $(6 \times 6 = 36)$.

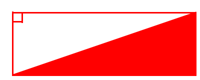
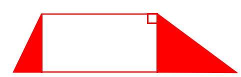
Example: Find the area of the rectangle shown below.



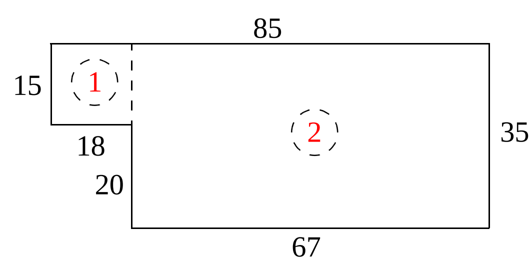
Area = LENGTH × HEIGHT = 20 × 9 = 180

FORMULAS TO FIND THE AREA OF CERTAIN SHAPES

AREA FORMULAS

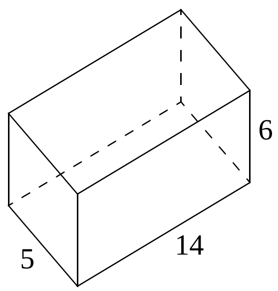
FIGURE	FORMULA	COMMENTS
Rectangle	bh	Base × Height (Also for the square)
Triangle	$\frac{bh}{2}$	Splitting the rectangle in two 
Trapezoid	$\frac{h(B_1 + B_2)}{2}$	Two bases. Forms from a rectangle and 2 triangles 
Circle	πr^2	Where r is the radius and $\pi = 3.14$

Example:
Find the area, in square feet, of the floor plan shown.
Areas of unusual shapes may be separated into familiar shaped areas that fit the equations known, then add them to find the answer.
The floor plan shown consists of two rectangles:
1. (15×18) and 2. (67×35). See diagram.

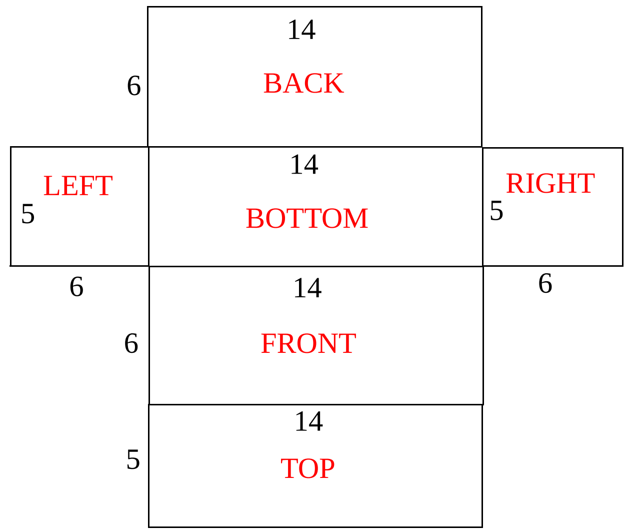


$$\text{Area}_{\text{rect.}} = \overset{1}{bh} + \overset{2}{bh} = \underset{1}{(15 \times 18)} + \underset{2}{(67 \times 35)} = 2615\ \text{ft}^2$$

Example: In the box shown below, the bottom has an area of 14 × 5 and the height is 6. If the outside of the box is to be painted, how much area will be covered with paint? (Units in inches)



Before any computation is performed, all surfaces must be laid out. The box is unfolded and all corresponding dimensions identified. The figure to the right shows this. Pairing the six sides of the box:



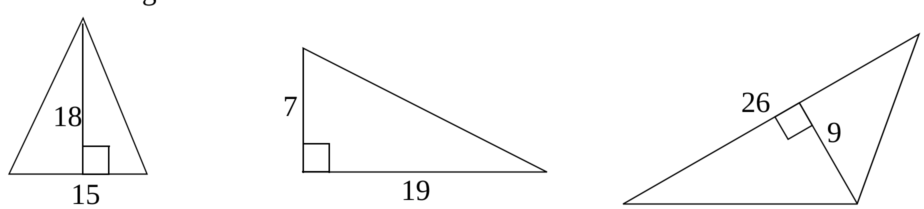
top & bottom — front & back — left & right

$$\text{Area}_{\text{rect.}} = 2(bh) + 2(bh) + 2(bh)$$

$$= 2(5 \times 14) + 2(6 \times 14) + 2(5 \times 6)$$

$$= 140 + 168 + 60 = 368 \text{ in}^2$$

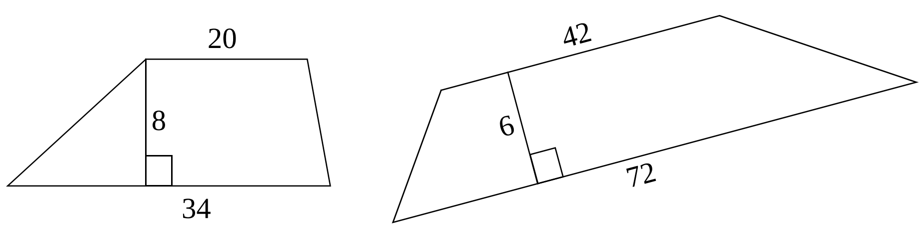
Example: Find the area of the triangles shown.



$$\text{Area}_T = \frac{bh}{2} \qquad \text{Area} = \frac{15 \times 18}{2} = 135 \qquad \text{Area} = \frac{19 \times 7}{2} = 66.5 \qquad \text{Area} = \frac{26 \times 9}{2} = 117$$

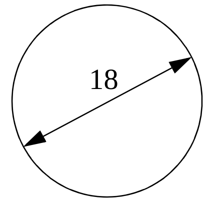
Notice that each triangle has three bases and three heights. Which one to use is optional. However, base and height must form a right (90º) angle.

Example: Find the area of the trapezoids shown.



$$\text{Area}_{\text{trap.}} = \frac{h(B_1 + B_2)}{2} \qquad \text{Area} = \frac{8(20 + 34)}{2} = 216 \qquad \text{Area} = \frac{6(42 + 72)}{2} = 342$$

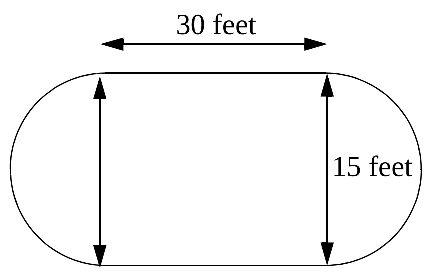
Example: Find the area, in square meters, of the circle shown.



Because the measure given in the diagram is the diameter and the radius is needed, divide diameter by 2 and calculate the answer.

$$\text{Area}_C = \pi r^2 = (3.14)(9^2) = (3.14)(81) = 254.34 \text{ m}^2$$

Example: Find the area, in square feet, of the compound figure shown.



The figure consists of two half-circles and a rectangle. Find the areas of half circles and rectangle separately, then combine.

Because the half circles are the same size, they'll be joined to make one complete circle with a diameter of 15 and a radius (half) of 7.5.

$$\text{Area} = bh + \pi r^2$$

$$\text{Area} = (30)(15) + (3.14)(7.5)^2$$

$$\text{Area} = 450 + 176.625 = 626.6 \text{ ft}^2 \text{ (rounded)}$$

Practice:

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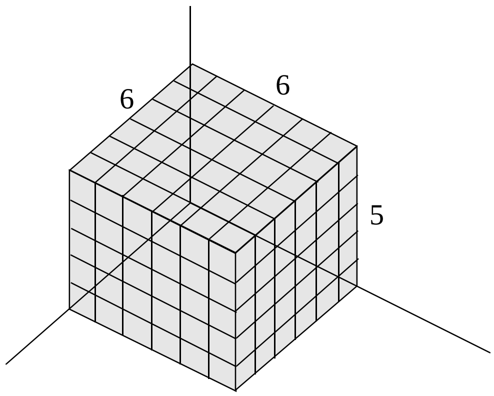
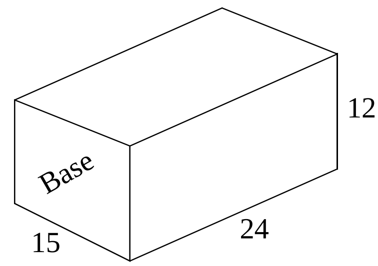
Section 12.5
Volume

Volume is the way *space* is measured. Space could be a room, a box, a warehouse, or the inside of an automobile engine. Volume is the product (multiplication) of quantities measured in THREE directions.



In the graph to the left, volume is the solid that has been darkened and it accounts for every one of the smaller cubes in the corner. In this particular example, the top layer has (6 × 6) = 36 cubes. Because there are 5 layers of 36 cubes each, the total volume is (36)(5) = 180.



Example: Find the volume, in inches, for the rectangular prism shown.

$\text{Volume}_{\text{prism}} = Bh$ Where*:* B = Area of base *h* = Height or length

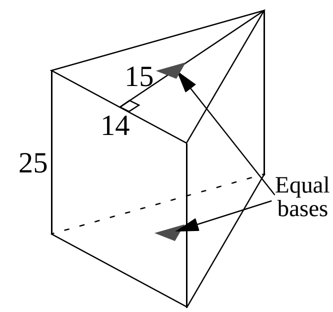
Volume = (15)(12)(24) = 4320

FORMULAS TO FIND THE VOLUME OF CERTAIN SHAPES

VOLUME FORMULAS

FIGURE	FORMULA	COMMENTS
Rectangular prism	Bh	Area of Base × Height or Length (Also for the cube.)
Triangular prism	$\frac{Bh}{2}$	The volume of any prism is the area of the cross-section of the prism times height or length.
Cylinder	$\pi r^2 h$	Where r is the radius, $\pi = 3.14$, and h the height or length.
Cone	$\frac{\pi r^2 h}{3}$	Volume of a cone is one-third the volume of a cylinder of the same height and diameter.
Pyramid	$\frac{Bh}{3}$	Volume of any pyramid is one-third the volume of a prism of the same height and base.
Sphere	$\frac{4}{3}\pi r^3$	Notice that the volume of the sphere involves the radius cubed (third power.)

Example: Find the volume, in cubic feet, of the triangular box shown.



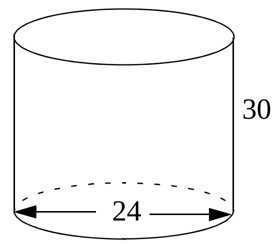
The box is in the shape of a triangular prism. Two *hs* are found: One for the triangular base (15), and the second one for the prism itself (25).

The area of the base (B) of the triangle is $\frac{bh}{2}$, where $b = 14$ and $h = 15$.

The h in the equation below refers to the height of the prism (25).

$$B = \frac{(14)(15)}{2} = 105 \qquad \text{Volume}_{\text{T. P.}} = \frac{Bh}{2} = \frac{(105)(25)}{2} = 2625\ \text{ft}^3$$

Example: Find the volume of the water tank shown if the height is 30 feet and the width 24 feet.



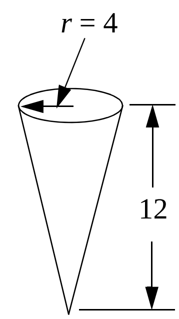
$\text{Volume}_{cy} = \pi r^2 h$ The width of a cylinder is the diameter.

Because the radius is needed, $r = \frac{24}{2} = 12$ feet

Base

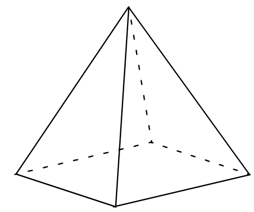
$$\text{Volume}_{cy} = \pi r^2 h = (3.14)(12)^2(30) = 13{,}564.8\ \text{ft}^3$$

Example: Find the volume of the ice cream cone shown if the radius is 4 centimeters and the height 12 centimeters.



$$\text{Volume}_{cone} = \frac{\pi r^2 h}{3} = \frac{3.14 \times 4^2 \times 12}{3} = 200.1\ \text{cm}^3 \text{ (rounded)}$$

Example: Find the volume of the pyramid shown if the height is 80 meters, and its base is 11,000 square meters.



$$\text{Volume}_{py} = \frac{Bh}{3} = \frac{11000 \times 80}{3} = 293{,}333.\overline{3}\ \text{m}^3.$$

Example: Find the volume inside a basketball if the ball is 9 inches wide.

In a sphere, width means diameter. Because the radius is needed, divide: $r = \frac{d}{2} = \frac{9}{2} = 4.5$

$$\text{Volume}_{sphere} = \frac{4}{3}\pi r^3 = \frac{(4)(3.14)(4.5)^3}{3} = 381.51\ \text{in}^3.$$

Practice:

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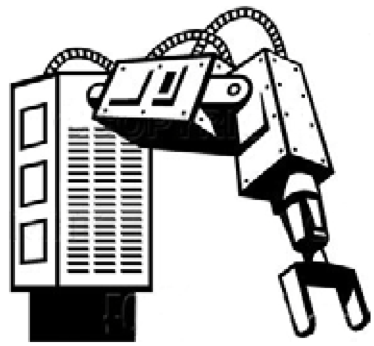
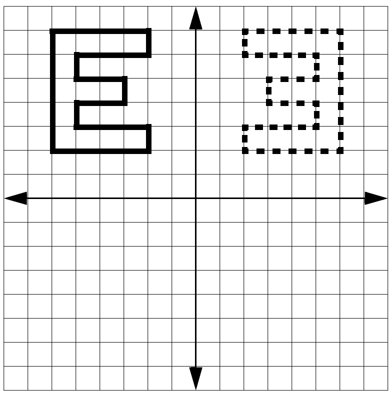
Section 12.6
Transformations: Reflections, Translations, and Rotations

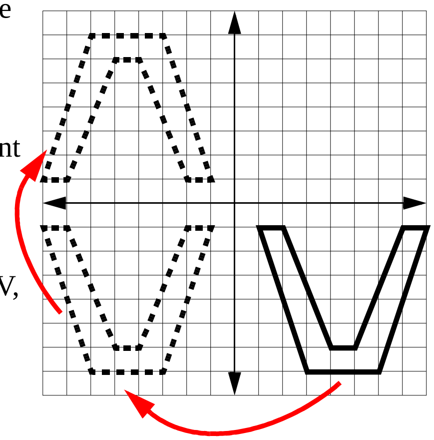
Transformations are movements that take place in a graph and, in more advanced models, are the basis for the programming that controls robots for industry and entertainment.



Industrial robot



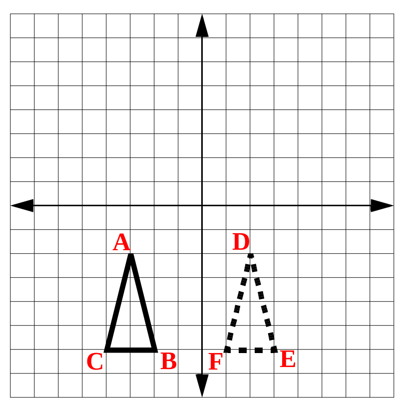
REFLECTIONS
A reflection is what the word implies: A mirror image of a figure or design. The graph to the left shows the reflection of an image from quadrant II to quadrant I. Notice that reflections must take place over a particular axis and turn out to be reversed. This particular reflection is in respect to the *y*-axis; therefore, every corresponding point of the letter "E" in quadrant II is at the same distance from the *y*-axis in quadrant I.



Example: Find the reflection of the letter "V" starting in quadrant IV, with respect to the *y*-axis and then the *x*-axis.

The solution can be seen in the graph to the right.

The points that define the letter "V" in the IV quadrant are, from left to right, (1,–1), (2,–1), (3,–7), (4,–6), (5,–6), (6,–7), (7,–1), and (8,–1). The reflection over the *y*-axis keeps the same values for *y*, but because the reflection is in over the *y*-axis, the *x* values change sign: (–1,–1), (–2,–1)...



The second reflection is going to quadrant II, where the *x* values are negative and the *y* values positive; therefore, again the values will be the same, except that the signs will be the opposite (–,+) to the original in quadrant IV.

TRANSLATIONS
Translations are movements which go UP, DOWN, RIGHT, and/or LEFT.

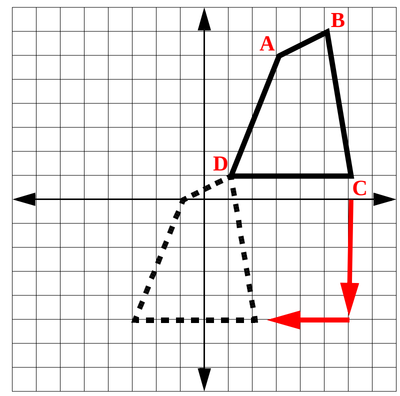
Example: In the graph to the right, make a translation of triangle (Δ)ABC five units to the right.

The original position of ΔABC was A(–3,–2), B(–2,–6), and C(–4,–6). Because it is to the right, all the *x* values of the coordinates change, but the *y* values stay the same. Simply add 5 to each *x*.

–3 + 5 = 2   –2 + 5 = 3   –4 + 5 = 1

The new coordinates are: D(2,–2), E(3,–6), and F(1,–6)

Example: In the graph to the right, make a translation of quadrilateral (□)ABCD four units to the left and six units down ($x – 4$, $y – 6$).
(Left, Down)



Because the translation is both to the LEFT and DOWN, both coordinates will change.

The original figure is at: A(3,6), B(5,7), C(6,1), D(1,1)

Translation to the left (*x-coordinate*) –4:

3 – 4 = –1   5 – 4 = 1   6 – 4 = 2   1 – 4 = –3

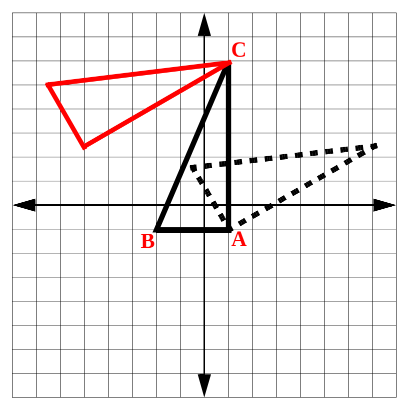
New coordinates after the first move: (–1,6), (1,7), (2,1), (–3,1)

Translation down (*y-coordinate*) –6:   6 – 6 = 0   7 – 6 = 1   1 – 6 = –5   1 – 6 = –5

New coordinates after the second move: (–1,0), (1,1), (2,–5), (–3,–5)

ROTATION

Measured in angles, rotation, also called *angular motion*, is the circling motion of a spinning object. All rotations must have a "center of rotation" about which the object moves.

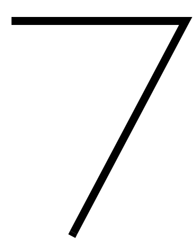


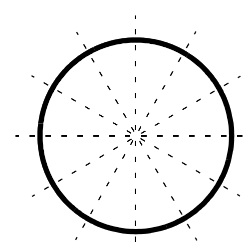
Example: In the graph to the right, rotate (rotation is always clockwise) the ΔABC 60º using point "A" as the center of rotation.

Because the rotation took place around point "A", point "A" did not move; however, both point "B" and "C" moved. In rotation, points farther away move more than points closer to the center of rotation; therefore, point "B" moved less than point "C." If the center of rotation changes, for example from "A" to "C", then the new location of the triangle will be completely different. (The red triangle above shows a rotation of 60º about "C")

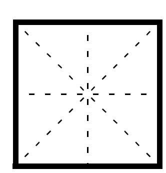
AXES (OR LINES) OF SYMMETRY

An object is said to have an axis of symmetry if, when folded about an axis, both sides of the fold match. In other words, there is "mirror symmetry" of half of the object. For example, a circle has an infinite number of axes of symmetry, while the number 7 has none.





Example: Find the number of axes of symmetry of a square.



The number of lines are 4, which is the number of axes. (Horizontal, vertical, and both diagonals.

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